Exam.Code:1014
Sub. Code: 7431

2123

M.E. (Mechanical Engineering)
First Semester
MME-101: Advanced Engineering mathematics

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, selecting atleast two questions from each Section. Use of Simple calculator is allowed. All questions carry equal marks.

x-x-x

SECTION-A

1. (a) Find the power series solution about $x = 0$, of the differential equation:

$$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 0.$$

(b) Find two linearly independent series solutions of differential equation:

$$x\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = 0 \text{ about } x = 0.$$

2. Explain the Legendre differential equation and locate its singularity. Also, find its solution.

3. (a) Reduce the differential equation: $x\frac{d^2y}{dx^2} + a\frac{dy}{dx} + k^2 x^r y = 0$ to standard Bessel's differential equation. Hence, solve: $x\frac{d^2y}{dx^2} + 75\frac{dy}{dx} + xy = 0$.

(b) Derive the generating function of the Bessel's polynomial.

4. (a) State and prove orthogonality of Legendre polynomials.

(b) Find the eigenvalues and eigenfunctions of the Sturm-Liouville problem:

$$\frac{d^2y}{dx^2} + \lambda y = 0,\ y(0) = y(2\pi),\ \frac{dy}{dx}(0) = \frac{dy}{dx}(2\pi).$$

SECTION-B

5. (a) Approximate y and z by using the Runge-Kutta method for the particular solution of simultaneous differential equations: $\frac{dy}{dx} = x + yz;\ \frac{dz}{dx} = xz + y$, given that $y = 1,\ z = -1$ when $x = 0$.

(b) Solve: $\frac{d^2y}{dx^2} = x\frac{dy}{dx} - y;\ y(0) = 3,\ \frac{dy}{dx}(0) = 0$ to approximate $y(0.1)$ by the Picard's method.

6. (a) Solve the boundary value problem: $\frac{d^2y}{dx^2} - 64y + 10 = 0, y(0) = y(1) = 0$ for $x = 0.5$, by the finite difference method.

(b) Write a note on the finite difference method for solving partial differential eequations. Obtain the forward and backward finite difference approximations to u_{xx}.

P.T.O.

7. Solve by relaxation method, the Laplace equation: $u_{xx} + u_{yy} = 0$ inside a square region bounded by the lines $x = 0, x = 4, y = 0, y = 4$ given that $u = x^2 y^2$.

8. (a) Use the Crank-Nicolson method to solve: $u_t = u_{xx}$ subject to the conditions:

$$u(x, 0) = \sin \pi x, 0 \leq x \leq 1, u(0, t) = u(1, t) = 0.$$

(b) Solve the wave equation: $u_{tt} = u_{xx}$ up to $t = 0.2$ with spacing 0.1 subject to the conditions: $u(0, t) = 0, u(1, t) = 0, u_t(x, 0) = 0, u(x, 0) = 1 =+ x(1 - x)$.

x-x-x