

Note: Attempt five questions in all. Question 1 is compulsory. Do any two questions each from Part A and Part B.

Q1)

(2 x 5 = 10)

- a) Arrange the following asymptotic complexities in increasing order? Justify your answer.  
 (i)  $f_1(n) = 2^n$     (ii)  $f_2(n) = n^{(3/2)}$     (iii)  $f_3(n) = n \cdot \log(n)$     (iv)  $f_4(n) = n^{\log(n)}$
- b) Compare memorization and dynamic programming.  
 c) Give a suitable example to justify the case where application of BFS is justified and computationally efficient over DFS.  
 d) Briefly discuss the fundamental principle behind solving the maxflow problem.  
 e) Give one real world application of Bipartite matching.

## PART-A

Q2)

- a) Explain the algorithm for Reversort with an example. (4)  
 b) Find out the cost of Reversort if the input array is [5, 2, 6, 8, 9, 7, 4, 3, 1] (4)  
 c) What is the time complexity of the function **fun** given below? (2)

```

1 int fun(int n)
2 {
3     int count = 0;
4     int i, j;
5     for(i = 0; i < n; i++)
6     {
7         j = 1;
8         while(j < n)
9         {
10            count += 1;
11            j = j * 2;
12        }
13    }
14    return count;
15 }
```

Q3)

Consider the following 5 items (one unit for each) with their weights and value. The task is to pick a subset of these items such that their total weight should be lesser or equal to 13 (maximum weight capacity  $C \leq 13$ ) and their total value is maximized.

Item No.	Weight(W)	Value(V)
1	7	28
2	11	55
3	2	34
4	4	32
5	9	60

Apply the greedy approach to solve the above problem assuming that each item has only one unit and it cannot be split. (10)

Q4)

Consider the 11 people labelled from 0 to 10 and the following list of friendships, where each pair  $(u,v)$  represents that  $u$  and  $v$  are friends on social network.

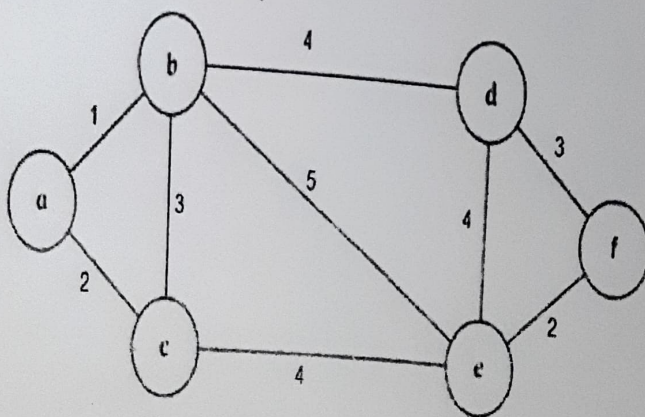
$[(0,1),(1,3),(0,3),(5,8),(6,10),(7,1),(7,0),(7,3),(9,4),(9,2),(4,2),(6,4)]$

If a friend circle is defined as a group of people who are directly or indirectly connected by friendships, then determine the number of friend circles in this network. Also name the strategy that will help in solving this problem. (10)

### PART-B

Q5)

- a) In a school, there are  $n$  teachers and  $2n$  subjects. Each teacher is required to teach exactly 2 subjects. However, teachers have their preferences for subjects they would like to teach, and the school wants to maximize overall satisfaction by assigning subjects to teachers based on their preferences. The preferences are modelled as a directed graph  $G$ , where there exists an edge from a teacher node  $T_i$  to a subject node  $S_j$  in  $G$  if teacher  $T_i$  prefers teaching subject  $S_j$ . How can this problem be modelled as a network flow problem? (3)
- b) Consider the graph  $G$  given below.



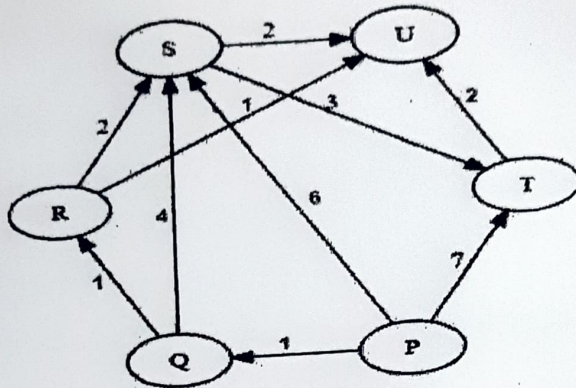
Let  $\alpha$  denote the number of minimum spanning trees of  $G$  and  $\beta$  denote the weight of such a minimum spanning tree. Find out the value of  $\alpha + \beta$ . (7)

Q6)

What is dynamic programming? Differentiate between top-down and bottom-up approaches of dynamic programming. Consider **Dice with combinations problem**, find out the number of ways to construct sum 10 by throwing a dice one or more times. (10)

Q7)

How Dijkstra's algorithm is different from other shortest path algorithms? Explain the algorithm/pseudo code of the same. If we try to find the shortest path from node P to all other nodes using **Dijkstra's algorithm**, for the following graph then in what order do the nodes get included in the visited set?



(10)