Exam. Code: 0927
Sub. Code: 6571

2123

B.E. (Electronics and Communication Engineering)
Third Semester
MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part

x-x-x

Question I (a) Define rank of a matrix. Give an example.

(b) Is translation of axes a linear transformation? Justify.

(c) When is a set of vectors said to be linearly independent? Prove that if any set of vectors contains the zero vector, then that set is necessarily linearly dependent.

(d) Check the analyticity of the function $f(z) = z^6$ defined on the whole complex plane using Cauchy-Riemann equations.

(e) Find every complex number z, which fulfills the equation $e^{2z+4i} = 3\sqrt{3} + 3i$.

(2 × 5 = 10)

Part A

Question II (a) Solve the following system of linear equations.

$$7x_1 + 2x_2 - 2x_3 - 4x_4 + 3x_5 = 8$$

$$-3x_1 - 3x_2 + 2x_4 + x_5 = -1$$

$$4x_1 - x_2 - 8x_3 + 20x_5 = 1$$

(b) Find the inverse of the following matrix using Gauss-Jordan method:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

(5+5=10)

Question III (a) Find the eigen values and eigen vectors of the following matrix A.

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 2 & 0 & -2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Is this matrix diagonalizable?

(b) State the Cayley-Hamilton theorem. Using it, invert the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}.$$

(5+5=10)

P.T.O.

Question IV (a) Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear mapping defined by $F(x, y, z, t) = (x - y + z + t, 2x - 2y + 3z + 4t, 3x - 3y + 4z + 5t)$. Find basis and dimension of image of F.

(b) Consider the following two basis of $\mathbb{R}^3$:
$E = \{e_1, e_2; e_3\} = \{(1,0,0), (0,1,0), (0,0,1)\}$
$S = \{u_1, u_2, u_3\} = \{(1,0,1), (2,1,2), (1,2,2)\}$
Find a change of basis matrix P from E to S and a change of basis matrix Q from S to E. What is the relation between P and Q?

(5+5=10)

Part B

Question V (a) When is a complex valued function $f(z)$ of a complex variable z said to be analytic? Find the regions in the complex plane where the following functions are analytic:
(i) $f(z) = | z |$ (ii) $f(z) = \text{Re } z/\text{Im } z$.

(b) Let u and v denote the real and imaginary components of the function f defined by the equations

$$f(z) = \begin{cases} \dfrac{(\overline{z})^2}{z} & \text{when } z \neq 0, \\ 0 & \text{when } z = 0. \end{cases}$$

Verify that the Cauchy-Riemann equations are satisfied at the origin $z = 0$ but $f'(0)$ nevertheless fails to exist.

(5+5=10)

Question VI (a) Give two Laurent series expansions in powers of z for the function $\dfrac{1}{z^2(1-z)}$ and specify the regions in which those expansions are valid.

(b) Show that the function $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2$ is harmonic. Also find its harmonic conjugate.

(5+5=10)

Question VII (a) Find the bilinear transformation which maps $z_1 = 0$, $z_2 = -\iota$, $z_3 = -1$ to the points $w_1 = \iota, w_2 = 1, w_3 = 0$ respectively.

(b) Use the transformation $w = e^z$ to map the rectangular region $a \leq x \leq b$, $c \leq y \leq d$ in xy plane onto a region in the uv plane.

(5+5=10)

x-x-x