Exam.Code:0933
Sub. Code: 6660

2123

B.E. (Electrical and Electronics Engineering)

Third Semester

BS-EE-305: MATH-III

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.

x-x-x

1. (a) Define an augmented matrix and row echelon form of a matrix. What is the purpose of row echelon form? How do you know if an augmented matrix is in row echelon form?

(b) Define a vector space and subspace with a suitable example. Prove that the intersection of two subspaces is itself a subspace of a vector space.

(c) Define eigenvalue problem of a matrix. The eigenvalues of the matrix A are 2, 3, 1. Then, find the eigenvalues of a matrix $A^{-1} + A^2$.

(d) List out any three differences and similarities between sin x and sin z.

(e) Define fixed points and critical points of a map. Find the same for $w = \frac{1+z}{1-z}$.

SECTION-A

2. (a) Solve the homogeneous system of equations:AX = 0, where $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & -2 & 3 \\ 1 & 5 & -4 \end{bmatrix}$.

Find the rank (A) and nullity (A). (3 + 3 + 4)

(b) Define linear combination of vectors. Show the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ is a linear combination of matrices: $M_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $M_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $M_3 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

(c) Prove that all upper triangular matrices of order 2 over R is a vector space with standard matrix addition and scalar multiplication.

3. (a) Find the eigenvalues and eigenvectors for the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

(b) Examine whether A is similar to B, where $A = \begin{bmatrix} 5 & 5 \\ -2 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$.

(c) Prove that the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable over the field C.

4. (a) Prove that the linear transformation T: $R^3 \to R^3$ defined by

$T(x, y, z) = (x, a\,y, z)$, where a is a fixed real number, is an isomorphism.

(b) Find a linear transformation T: $R^3 \to R^2$ such that $T(1, 2, 3) = (1, 1)$ and $T(0, 1, 2) = (1, 2)$.

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(c) Let $V = R^2$ with bases $B = \{(1,1), (1, -1)\}$ and $B^1 = \{(2, -1), (-1, 1)\}$.

Find the transition matrix from B to B^1 and vice-versa.

SECTION-B

5. (a) Define a bounded function of a complex variable. Prove that $w = \cos z$ is not a bounded function.

(b) Prove that the function $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.

(c) Prove that $f(z) = \cosh z$ is an analytical function using C-R equations.

6. (a) State Laurent's theorem. Find Laurent's expansion about the singularity $z = 1$ for the function $f(z) = \frac{z^2}{(z-1)^2 (z+3)}$.

(b) Using Cauchy residue theorem, evaluate $\oint \frac{z^2}{(z-1)^2 (z+2)}$, where C is $|z - 2| = 2$.

7. (a) Evaluate $\int_0^{\pi} \frac{3\, d\theta}{9 + \sin^2 \theta}$ using complex integration.

(b) Discuss the map $w = z^2$.

x-x-x