Exam.Code:0921
Sub. Code: 6471

2122
B. E. (Information Technology)
Third Semester
ASM-301: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.

x-x-x

Attempt the following:-

1 (a) Find the span of z-axis and the plane $x + z$ in V_3. (1.5)

(b) Can $(2, 7, 8)$ be written as a linear combination of vectors $(1, 2, 3)$, $(1, 3, 5)$ and $(1, 5, 9)$. Justify your answer. (2)

(c) Let V be a vector space of functions $f : \mathbb{R} \longrightarrow \mathbb{R}$. Check whether $W = \{f(x) : f(-x) = f(x)\}$ is a subspace of V or not. (1.5)

(d) A fair die is tossed 300 times. Find the expected number E and the standard deviation σ of the number of 2's. (1.5)

(e) Define (a) mutually exclusive events (b) equally likely events (c) exhaustive events. (1.5)

(f) Let A and B be events with $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$, and $P(A \cap B) = \frac{1}{4}$. Find $P(A|B)$ and $P(B|A)$. (2)

Section-A

2 (a) Determine a linear transformation $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ whose range space is

$$\{(1, 2, 0), (0, 1, 1) (1, 3, 1)\}.$$

(5)

(b) Let $V = \mathbb{R}^3$ be a vector space and $W_1 = \{(a, b, c, d, e) \in \mathbb{R}^3;\ a + c - 3d + e = 0\}$ and $W_2 = \{(a, b, c, d, e) \in \mathbb{R}^3;\ b - c - e = 0 \text{ and } a = 4\}$. Find a basis for $W_1 \cap W_2$. (5)

3 (a) Is the matrix $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 8 \end{pmatrix}$ diagonalizable? Prove or disprove. (5)

Contd....P/2

(b) Solve the system of linear equations

$$x + 2y + 3z = 1,\ 2x + 3y + z = 1,\ x + 2y + 2z = 1,$$

if consistent. (5)

4 (a) For the linear map $F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ defined by

$$F(x, y, z) = (2x - 7y - 4z, 3x + y + 4z, 6x - 8y + z),$$

find a vector $(x, y, z) \in \mathbb{R}^3$ such that $F(x, y, z) = (-6, 1, -5)$. (5)

(b) For the basis $B = \{x^3+x^2,\ x^2+x,\ x+1,\ 1\}$ of the vector space $P_3(x)$, of all polynomials of degree ≤ 3, find the coordinate vector of $2x^3 + x^2 - 4x + 2$ relative to the basis B. (5)

Section-B

5 (a) A committee of four is selected at random from a class with 12 students of whom seven are boys. Find the probability that the committee contains: (i) at least two boys, (ii) exactly two boys. (3)

(b) Let $X \sim N(\mu, \sigma^2)$. Find its moment generating function $M_X(t)$ and variance $V(X)$. (4)

(c) Write down the probability function of a Poisson variable. Show that it satisfies the properties of being a probability function. (3)

6 (a) Suppose two percent of the people on the average are left-handed. Find the probability of three or more left-handed among 100 people. (3)

(b) A box contains 10 coins where 5 coins are two-headed, 3 coins are two tailed, and 2 are fair coins. A coin is chosen at random and tossed. If a head appears, find the probability that the coin is fair. (3)

(c) A fair coin is tossed three times. Let X equals 0 or 1 accordingly as a head or a tail occurs on the first toss, and let Y equals the total number of heads that occurs. Find $\text{cov}(X, Y)$, and covariance of X and Y. (4)

7 (a) In a certain country, the heights of men are normally distributed with mean 175 cm and standard deviation 5 cm and the heights of women are normally distributed with mean 165 and standard deviation 6 cm. Find the probability that the mean height of three women chosen at random is greater than the mean height of four men chosen at random from the population. (5)

(b) A, B, C play a game and the chances of their winning it in an attempt are $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{1}{4}$, respectively. A has the first chance, followed by B and then by C. This cycle is repeated till one of them wins the game. Find their respective chances of winning the game. (5)

x-x-x