(i) Printed Pages: 4 Roll No. ....

(ii) Questions : 7 **Sub. Code :** 6 1 9 3

Exam. Code : 0 9 0 5

B.Engg. 1st Year (1st Semester)

(2123)

CALCULUS

(Common to All Streams)

Pape : ASM–101

Time Allowed : Three Hours] [Maximum Marks : 50

Note :—Attempt **FIVE** questions in all, including Question No. **1** which is compulsory and selecting **TWO** questions from each Section.

SECTION—A

1. Answer the following :

(a) Find limit of the sequence $a_n = n - \sqrt{n^2 - n}$.

(b) If $x^2 + y^2 = r^2$ and $x = r\cos\theta$ then find $\left(\frac{\partial x}{\partial r}\right)_\theta$ and $\left(\frac{\partial r}{\partial x}\right)_y$.

(c) Find the arc length for the curve $\frac{x^3}{12} + \frac{1}{x}, 1 \le x \le 4$, taking point $\left(1, \frac{13}{12}\right)$ as the starting point.

(d) Integrate $f(x, y, z) = \sqrt{x^2 + z^2}$ over the circle $\tilde{r}(t) = (a \cos t)\hat{j} + (a \sin t)\hat{k}, 0 \le t \le 2\pi$.

(e) Change the Cartesian integral in polar integral and evaluate :

$$\int_0^2 \int_0^x y \, dy \, dx.$$

5×2=10

SECTION—B

2. (a) Which tests is better, Ratio Test or Root Test ? Justify your answer by providing suitable example.

(b) Find the interval and radius of convergence for $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{2n+1}}{2n+1}$ and also find the values of x for which series converges absolutely and conditionally.

(c) For what values of p the series $\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$ converges ?

3+4+3

3. (a) Find the areas of the surface generated by revolving the curve $x = 2\sqrt{4-y}; 0 \le y \le \frac{15}{4}$ about the y-axis.

(b) Given a function $f(x, y, z) = \frac{x+y+z}{x^2+y^2+z^2+1}$, and a positive number $\epsilon = 0.015$. Show that there exists a $\delta > 0$ such that for all (x, y, z),

$$\sqrt{x^2+y^2+z^2} < \delta \Rightarrow |f(x, y, z) - f(0, 0, 0)| < \epsilon.$$

(c) Let $x = r\cos\theta$, $y = r\sin\theta$ in a differentiable function $w = f(x, y)$. Show that :

$$(f_x)^2 + (f_y)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2$$

3+3+4

4. (a) Maximize the quantity $f(x, y, z) = \dfrac{5xyz}{x + 2y + 4z}$ subject to constraint $xyz = 8$.

(b) A delivery company accepts only rectangular boxes, the sum of whose length and girth (perimeter of a cross-section) does not exceed 108 cm. Find the dimensions of an acceptable box of largest volume.

(c) A company manufactures stainless steel right circular cylindrical molasses storage tanks that are 25 ft high with a radius of 5 ft. How sensitive are the tanks' volumes to small variations (change) in height and radius ?

4+3+3

SECTION—C

5. (a) Reverse the order of integration, and evaluate the integral

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2 + y^2}}\, dx\, dy$$

(b) Find the volume of the first octant bounded by the co-ordinate planes, the plane $x + y = 4$ and the cylinder $y^2 + 4z^2 = 16$.

(c) Use the transformation $u = x - y$ and $v = 2x + y$ for x and y to evaluate the integral

$$\iint_R (2x^2 - xy - y^2)\, dx\, dy$$

for the region R in the first quadrant by the lines $y = -2x + 4$, $y = -2x + 7$, $y = x - 2$, and $y = x + 1$.

3+3+4

6. (a) Find the length of the curve

$$\vec{r}(t) = (\sqrt{2}t)\hat{i} + (\sqrt{2}t)\hat{j} + (1 - t^2)\hat{k}$$

from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$.

(b) Find the tangential and normal component of the acceleration $(\vec{a} = a_T T + a_N N)$ for the space curve

$$\vec{r}(t) = (a \cos t)\hat{i} + (a \sin t)\hat{j} + b\, t\, \hat{k}, \quad a, b \geq 0, a^2 + b^2 \neq 0$$

(c) Find the directional derivative of $f(x, y, z) = xyz$ in the direction of the velocity vector of the helix

$$\vec{r}(t) = (\cos 3t)\hat{i} + (\sin 3t)\hat{j} + 3t\, \hat{k},$$

at $t = \dfrac{\pi}{3}$.

3+4+3

7. (a) Show that the vector field

$$\int_C \vec{F} = (e^x \cos y + yz)\hat{i} + (xz - e^x \sin y)\hat{j} + (xy + z)\hat{k}$$

is conservative over its natural domain and find a potential function for it.

(b) State and verify Stoke's theorem for the hemisphere $S : x^2 + y^2 + z^2 = 9$, $z \geq 0$, its bounding circle $C : x^2 + y^2 = 9$, $z = 0$, and the vector field $\vec{F} = y\hat{i} - x\hat{j}$.

5+5