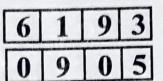
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Roll No.

(i) (ii)

Questions : 7

Sub. Code : 6



Exam. Code : 0

B.Engg. 1st Year (1st Semester)

(2123)

CALCULUS

(Common to All Streams)

Pape : ASM-101

Time Allowed : Three Hours] [Maximum Marks : 50

Note :—Attempt FIVE questions in all, including Question No. 1 which is compulsory and selecting TWO questions from each Section.

SECTION—A

1. Answer the following :

(a) Find limit of the sequence $a_n = n - \sqrt{n^2 - n}$.

(b) If $x^2 + y^2 = r^2$ and $x = r \cos \theta$ then find $\left(\frac{\partial x}{\partial r}\right)_{\theta}$ and

 $\left(\frac{\partial \mathbf{r}}{\partial \mathbf{x}}\right)$.

(c) Find the arc length for the curve $\frac{x^3}{12} + \frac{1}{x}$, $1 \le x \le 4$,

taking point $\left(1, \frac{13}{12}\right)$ as the starting point.

[Turn over

- (d) Integrate $f(x, y, z) = \sqrt{x^2 + z^2}$ over the circle $\tilde{r}(t) = (a \cos t)\hat{j} + (a \sin t)\hat{k}, 0 \le t \le 2\pi$.
- (e) Change the Cartesian integral in polar integral and evaluate :

$$\int_{0}^{x} \int_{0}^{2} y \, dy \, dx. \qquad 5 \times 2 = 10$$

SECTION-B

- (a) Which tests is better, Ratio Test or Root Test ? Justify your answer by providing suitable example.
 - (b) Find the interval and radius of convergence for

 $\sum_{n=0}^{\infty} \frac{(-1)^n (x-1)^{2n+1}}{2n+1}$ and also find the values of x for

which series converges absolutely and conditionally.

(c) For what values of p the series
$$\sum_{n=1}^{\infty} \frac{\ln n}{n^p}$$
 converges ?
3+4+3

- 3. (a) Find the areas of the surface generated by revolving the curve $x = 2\sqrt{4 y}$; $0 \le y \le \frac{15}{4}$ about the y-axis.
 - (b) Given a function $f(x, y, z) = \frac{x + y + z}{x^2 + y^2 + z^2 + 1}$, and a

positive number $\epsilon = 0.015$. Show that there exists a $\delta > 0$ such that for all (x, y, z),

$$\sqrt{x^2 + y^2 + z^2} < \delta \Rightarrow |f(x, y, z) - f(0, 0, 0)| < \epsilon.$$

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(c) Let $x = r \cos \theta$, $y = r \sin \theta$ in a differentiable function w = f(x, y). Show that :

$$(f_x)^2 + (f_y)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta}\right)^2$$
 3+3+4

4. (a) Maximize the quantity $f(x, y, z) = \frac{5xyz}{x + 2y + 4z}$ subject to constraint xyz = 8.

- (b) A delivery company accepts only rectangular boxes, the sum of whose length and girth (perimeter of a crosssection) does not exceed 108 cm. Find the dimensions of an acceptable box of largest volume.
- (c) A company manufactures stainless steel right circular cylindrical molasses storage tanks that are 25 ft high with a radius of 5 ft. How sensitive are the tanks' volumes to small variations (change) in height and radius ? 4+3+3

SECTION-C

5. (a) Reverse the order of integration, and evaluate the integral

$$\int_{0}^{1} \int_{x}^{\sqrt{2-x^{2}}} \frac{x}{\sqrt{x^{2}+y^{2}}} \, dx \, dy$$

(b) Find the volume of the first octant bounded by the co-ordinate planes, the plane x + y = 4 and the cylinder $y^2 + 4z^2 = 16$.

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Turn over

(c) Use the transformation u = x - y and v = 2x + y for x and y to evaluate the integral

 $\iint_{\mathbb{R}} \left(2x^2 - xy - y^2 \right) dx \, dy$

for the region R in the first quadrant by the lines y = -2x + 4, y = -2x + 7, y = x - 2, and y = x + 1. 3+3+4

6. (a) Find the length of the curve

$$\bar{\mathbf{r}}(t) = (\sqrt{2}t)\hat{\mathbf{i}} + (\sqrt{2}t)\hat{\mathbf{j}} + (1-t^2)\hat{\mathbf{k}}$$

from (0, 0, 1) to $(\sqrt{2}, \sqrt{2}, 0)$.

(b) Find the tangential and normal component of the acceleration $(\vec{a} = a_T T + a_N N)$ for the space curve

 $\bar{r}(t) = (a \cos t)\hat{i} + (a \sin t)\hat{j} + b t \hat{k}, a, b \ge 0, a^2 + b^2 \ne 0$

(c) Find the directional derivative of f(x, y, z) = xyz in the direction of the velocity vector of the helix

$$\vec{r}(t) = (\cos 3t)\vec{i} + (\sin 3t)\vec{j} + 3t \ k$$
,

at
$$t = \frac{\pi}{3}$$
. $3+4+3$

7. (a) Show that the vector field

 $\int_{C} \vec{F} = (e^{x} \cos y + yz)\hat{i} + (xz - e^{x} \sin y)\hat{j} + (xy + z)\hat{k}$ is conservative over its natural domain and find a potential function for it.

(b) State and verify Stoke's theorem for the hemisphere S : x² + y² + z² = 9, z ≥ 0, its bounding circle C : x² + y² = 9, z = 0, and the vector field F = yî - xĵ. 5+5

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