1/7/22 (Evening)

Exam.Code:0918
Sub. Code: 6793

2062

B.E. (Computer Science and Engineering)
Sixth Semester
CS-602: Linear Algebra and Probability Theory

Time allowed: 3 Hours | Max. Marks: 50

***NOTE:** Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.*

x-x-x

Q.No:1 (a) Does the set of all lower triangular matrices of order n over $\mathbb{R}$ form a vector space over $\mathbb{R}$ or not with respect to usual addition and scalar multiplication of matrices? Justify.

(b) Let V be the vector space of polynomials of degree ≤ 3 over $\mathbb{R}$. Discuss if the vectors $v_1 = t^3 - 3t^2 + 5t + 1$, $v_2 = t^3 - t^2 + t + 1$, $v_3 = 5t^3 + 8t^2 + t + 3$ are linear independent or linearly dependent?

(c) Let $T : U :\rightarrow V$ be a linear transformation. Prove that kernel of T is a subspace of U.

(d) The probability that a teacher will give surprise test during any class is 3/5. If the student is absent on two days, what is the probability that he will miss atleast one test?

(e) If X is binomially distributed with parameters n and p, find the moment generating function of $Y = 3n + X$. (2 × 5)

Section: A

Q.No:2 (a) Extend $\{(-1, 2, 5)\}$ to two different basis of the vector space $\mathbb{R}^3(\mathbb{R})$.

(b) Prove that the union of two subspaces of a vector space V over a field F is a subspace of V if and only if one is contained in the other.

Q.No:3 (a) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be linear transformation defined by $T(x, y, z) = (x + 2y, y - z, x + 2z)$. Verify Rank Nullity theorem for T.

(b) For the matrix

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

find all eigen values. Is A diagonalizable? If yes, find the matrix P such that $P^{-1}AP$ is diagonal matrix.

P.T.O.

Q.No:4 (a) Let T be a linear operator on $\mathbb{R}^2$ defined by $T(x, y) = (4x - 2y, 2x + y)$. (i) Find the matrix of T relative to the basis $B = \{(1, 1); (-1, 0)\}$. (ii) Also verify that any vector $v \in \mathbb{R}^2$, $[T; B][v; B] = [T(v); B]$.

(b) Let T be a linear operator on $\mathbb{R}^3$ defined by $T(x, y, z) = (x, \lambda y, z)$, where λ is a fixed non-zero real number. Prove that T is an isomorphism.

Section: B

Q.No:5 (a) Discuss the properties of marginal and conditional distributions.

(b) State and prove Baye's theorem.

Q.No:6 (a) Show that in a Poisson distribution with unit mean, mean deviation about mean is $(2/e)$ times the standard deviation.

(b) If X is uniformly distributed on (0, 20), find its cumulative distribution function.

Q.No:7 (a) In a normal distribution, 7% items are below 35 and 89% are below 63. Find mean and standard deviation of the distribution.

(b) Prove that the coefficient of correlation is independent of change of scale and origin.

x-x-x