Exam.Code:0922
Sub. Code: 6605

2062
B.E. (Information Technology)
Fourth Semester
ASM-401: Discrete Structure

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.*

x-x-x

I)

a. Give an example of a connected graph that has neither a Hamiltonian cycle nor an Euler circuit. Hence find its chromatic number. **(2)**

b. Let $S = \{2,4,5,10,15,20\}$. Consider the partial order '$\leq$' defined by divisibility relation on S. Draw the Hasse diagram for the poset $(S, \leq)$. Find all maximal elements of this poset. **(2)**

c. What is an Eulerian path? What is the necessary and sufficient condition for a connected graph to have an Eulerian path? **(2)**

d. From a deck of 52 playing cards, how many must be picked up so that at least one of them is a heart. **(2)**

e. Is the following argument valid? Give reason. **(2)**

$$\begin{array}{l} p \\ p \vee q \\ q \to r \to s \\ t \to r \\ \hline \therefore \neg s \to \neg t \end{array}$$

Part A

II)

a. Let $\mathscr{R}$ be the relation on the set $A = \{1, 2, 3, 4, 5\}$ such that the matrix of $\mathscr{R}$ is

$$M_{\mathscr{R}} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

. Suppose $\mathscr{R}^+ = (cl_{sym}(\mathscr{R})) \cup (cl_{tran}(\mathscr{R}))$, where '$cl_{sym}(\mathscr{R})$' and '$cl_{tran}(\mathscr{R})$' are symmetric and transitive closures of $\mathscr{R}$, respectively. Draw the digraph of $\mathscr{R}^+$ and show that it is an equivalence relation. **(7)**

b. Find the total number of one-one functions from the set A to the set $A \times B$ if A contains 3 elements and B contains 5 elements. **(3)**

III)

a. Prove or disprove that: **(5)**

i. $(p \to q) \vee (\neg p \to q)$ and q are logically equivalent.

ii. $(p \to q) \leftrightarrow (\neg p \vee q)$ is a contradiction.

P.T.O.

b. Express the following using predicates, quantifiers and logical connectives. Also test the validity of the consequences. **(5)**

i. All athletes are healthy. All healthy people do exercise. Vitamins are taken by all exercisers. Gopal is an athlete. Therefore, Gopal takes vitamins.

ii. If Rita works hard and has talent, then she will get a job. If she gets a good job, then she will be happy. Hence if Rita is not happy, then she didn't work hard or she doesn't have talent.

IV)

a. Define distributive Lattice. Prove that every distributive lattice is a modular lattice. **(4)**

b. Prove that the following statements are equivalent: **(6)**

i. n is an odd integer.

ii. $7n + 8$ is an odd integer

iii. n^2 is an odd integer.

Part B

V)

a. In an examination a student is asked to answer any 7 out of 10 questions. In how many ways can this be done? Out of these ways how many ways are there in which the student answers 7 questions out of 10 selecting at least 3 from the first 5 questions. **(3)**

b. A group of 123 workers went to a canteen for cold drinks, ice-cream and tea, 42 workers took ice-cream, 36 tea and 30 cold drinks. 15 workers purchased ice-cream and tea,. 10 ice-cream and cold drinks, and 4 cold drinks and tea but not ice-cream, 11 took ice-cream and tea but not cold drinks. Determine how many workers did not purchase any things. **(3)**

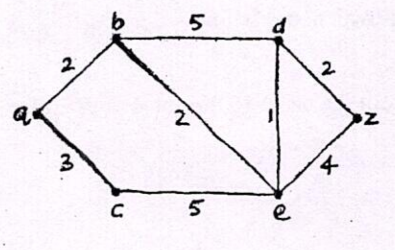
c. State and prove Euler's formula for connected planar graph. **(4)**

VI)

a. Define order and degree of a recurrence relation. Solve the following recurrence relation $a_r - 6a_{r-1} + 9a_{r-2} = r \cdot (3^r)$. **(4)**

b. If G is loop-free undirected graph with at least one edge, prove that G is bipartite iff its chromatic number is 2. **(4)**

c. Find shortest path from a to z each in the graph given below: **(2)**



VII)

a. State and prove LAGRANGE'S theorem for finite groups. **(5)**

b. Define field. If $D = a + b\sqrt{5} : a, b \in Z$, show that $(D, +, \cdot)$ is an integral domain. **(5)**

x-x-x