Exam.Code:1015
Sub. Code: 7760

2072
M.E. (Mechanical Engineering)
Second Semester
MME-201: Continuum Mechanics

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, selecting atleast two questions from each Section. Use usual notations and symbols for derivations. Assume suitable missing data if any. All questions carry equal marks.*

x-x-x

Section A

Q.1 Verify in direct notation that $(\mathbf{ST})^{-1} = \mathbf{S}^{-1}\mathbf{T}^{-1}$, and also prove that $(\mathbf{S}^{-1})^T = (\mathbf{S}^T)^{-1} \equiv \mathbf{S}^{-T}$.

Q.2 For the tensor:

$$\mathbf{T} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}.$$

Find the eigenvalues and eigenvectors. Also find the scalar invariants and then verify that the eigenvalues obtained by solving $\lambda^3 - I_1\lambda^2 + I_2\lambda - I_3 = 0$ are identical.

Q.3 In direct notation prove that $\dot{\mathbf{F}} = \mathbf{LF}$.

Q.4 Determine the covariant basis $\{\mathbf{g}_i\} = \{\mathbf{g}_1, \mathbf{g}_2, \mathbf{g}_3\}$ and the Christoffel symbols Γ^i_{jk} of the cylindrical polar coordinate system $(\theta^1 = r, \theta^2 = \theta, \theta^3 = z)$.

Section B

Q.5 Derive the Cauchy stress formula.

Q.6 Prove the Transport theorem by showing that

$$\frac{d}{dt}\int_{\mathcal{P}} \tilde{\phi}(\mathbf{x}, t)dv = \int_{\mathcal{P}} (\dot{\phi} + \phi\, div\mathbf{v})dv,$$

where ϕ is a scalar valued function.

Q.7 Discuss how a proposed constitutive equation for a specific material in the thermo-mechanical theory must satisfy the following:

i. conservation of angular momentum,
ii. invariance requirements under superposed rigid body motion,
iii. Clausius-Duhem inequality for all thermo-mechanical processes, and
iv. material symmetry requirements.

Q.8 Given the velocity field $v_1 = -c(x_1 + x_2)$, $v_2 = c(x_2 - x_1)$, $v_3 = 0$ with $c = 1\text{s}^{-1}$ for a Newtonian liquid with viscosity $\mu = 0.928\text{mPa·s}$. Find for a plane whose normal is in the $\mathbf{e_1}$ direction the excess of the total normal copressive stress over the pressure p, and also find the magnitude of the shearing stress.

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