Exam.Code:0906
Sub. Code: 6690

2072

B.E., Second Semester

MATHS-201: Differential Equations and Transforms

(Common to all streams)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.*

x-x-x

Question I (a) Solve by regrouping the terms the differential equation $(x^3 + xy^2 + y)\ dx + (y^3 + x^2y + x)\ dy = 0$

(b) Solve the differential equation: $(D^2 - 1)y = x \cos x$.

(c) Find the Laplace Transform of $te^{at} \sin at$

(d) Write the formula for the Fourier coefficients of a periodic function with period 2π.

(e) Write the general forms of the heat and wave equations.

$(2 \times 5 = 10)$

Part A

Question II (a) Solve the following differential equations:

(i) $[(x+1)^4 + 2\sin y^2]\ dx - 2y(x+1)\cos y^2\ dy = 0$

(ii) $(D^2 + 4D + 3)y = e^{-x} \sin x + x$

(b) Given that $y = e^{2x}$ is a solution of $(2x+1)\dfrac{d^2y}{dx^2} - 4(x+1)\dfrac{dy}{dx} + 4y = 0$, find the complete solution by reducing the order of the given differential equation.

(5+5=10)

Question III (a) Find a power series solution solution in powers of $(x-1)$ of the initial value problem $x\dfrac{d^2y}{dx^2} + \dfrac{dx}{dy} + 2y = 0,\ y(1) = 2,\ y'(1) = 4.$

(b) Find the general solution of following non-homogeneous differential equation using method of Variation of Parameters:

$(D^2 + 6D + 9)y = 16\dfrac{e^{-3x}}{x^2+1}$

(5+5=10)

Question IV (a) Find the Inverse Laplace Transforms of:

(i) $\dfrac{s}{s^4 + 4a^4}$

(ii) $\tan^{-1}\left(\dfrac{2}{s^2}\right)$

(b) Solve using Laplace transform $ty'' + 2y' + ty = \cos t,\ y(0) = 1$

(5+5=10)

P.T.O.

Part B

Question V (a) Find the Fourier series of the following function defined by

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases} \quad \text{with } f(x+2\pi) = f(x)\ \forall\, x \in \mathbb{R}.$$

Hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots = \frac{\pi}{4}$$

(b) Show that the given integral represents the indicated function:

$$\int_0^\infty \frac{\sin w \cos xw}{w}\, dw = \begin{cases} \pi/2 & \text{if } 0 \le x < 1 \\ \pi/4 & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$

(5+5=10)

Question VI (a) Solve the partial differential equation $(y + zx)p - (x + yz)q = x^2 - y^2$.

(b) (i) Eliminate the arbitrary constants from the equation $x^2 + y^2 + (z - a)^2 = b^2$.

(ii) Eliminate the arbitrary function f from the equation $f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.

(5+5=10)

Question VII Solve the one-dimensional wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$ subject to the boundary conditions $y(0,t) = y(L,t) = 0$ and initial conditions $y(x,0) = f(x)$, $\frac{\partial y}{\partial t}(x,0) = g(x)$ where $f(x)$ is the initial deflection and $g(x)$ is the initial velocity. Here $y = y(x,t)$. Also find the solution when $f(x) = \begin{cases} \frac{2k}{L}x & \text{if } 0 < x < L/2 \\ \frac{2k}{L}(L - x) & \text{if } L/2 < x < L \end{cases}$ and the initial velocity is 0.

(10)

x-x-x