Exam.Code:0906
Sub. Code: 6660

2072

B.E. (Bio-Technology) Second Semester

ASM-201: Differential Equations and Transforms

(Common to all streams)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each section. All questions carry equal marks.*

x-x-x

1. (a) Define the order and degree of a ordinary differential equation. Find the same for differential equation: $x\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^{-1} = y^2$.

   (b) Find a homogeneous linear ODE for which e^{-x} and e^{-2x} are the solutions.

   (c) Define Laplace and inverse Laplace transform. If $f(t) = e^t$ on $[0, \infty)$, then prove that $L(e^t)$ converges for $Re(s) > 1$.

   (d) Define Fourier series and Fourier transform. State the conditions for which a function f(x) can be represented as a Fourier series.

   (e) State one dimensional heat equation with boundary conditions and initial conditions for solving it.

SECTION-A

2. (a) Define exact differential equation. Solve the differential equation:

$$y(xy + 2x^2y^2)\,dx + x(xy - x^2y^2)\,dy = 0.$$

   (b) What is meant by variation of parameters? Solve differential equation by method of variation of parameters: $y^{11} - y = \frac{2}{1+e^x}$.

3. (a) Solve in power series about x = 0 of $(1+x^2)y^{11} + xy^1 - y = 0$.

   (b) Find inverse Laplace transform of $f(s) = \frac{s\,e^{-\frac{s}{2}} + \pi e^{-s}}{s^2+\pi^2}$ in terms of unit step function.

4. (a) Using convolution theorem, solve the IVP:

$$y^{11} + 9y = \sin 3t, y(0) = 0,\ y^1(0) = 0.$$

   (b) Use the Laplace transform to solve the IVP:

$$y^{11} + 4y^1 13y = e^{-t}, y(0) = 0, y^1(0) = 2.$$

P.T.O.

SECTION-B

5. (a) Obtain the Fourier series expansion of $f(x) = x \sin x$ in $(-\pi, \pi)$.

   (b) Find the Fourier cosine and sine transforms of $e^{-ax}, a > 0$ and hence deduce their inversion formulae.

6. (a) Obtain the PDE governing the equation: $\varphi(u, v) = 0$, where

   $u = xyz,\ v = x + y + z$.

   (b) Find the general solution of a PDE: $(3 - 2yz)p + x(2z - 1)q = 2x(y - 3)$. Hence, obtain the particular solution which passes through the curve:

   $z = 0, x^2 + y^2 = 4$.

7. (a) Using D' Alembert solution, solve: $y_{tt} = 4\, y_{xx}, 0 < x < 1, t > 0,$

   $y(0, t) = y(\pi, t) = 0, y(x, 0) = \sin x$ and $y_t(x, 0) = \sin x$.

   (b) Using the method of separation of variables, solve:

   $$3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(x, 0) = 4\,e^{-x}.$$

x-x-x