July 2022 (Set-1)

Exam.Code:0905
Sub. Code: 6631

2072

B.E. (Bio-Technology) First Semester

MATHS-101: Calculus

(Common with Second Semester)

(Common to all streams)

Time allowed: 3 Hours

Max. Marks: 50

***NOTE:** Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.*

x-x-x

Question I (a) Prove that if a series $\sum_{n=1}^{\infty} a_n$ converges, then the sequence $\{a_n\}$ converges to 0.

(b) Find the area enclosed by the cardioid $r = a(1 + \cos\theta)$.

(c) Find the length of the arc of the parabola $x^2 = 4y$ measured from the vertex to one extremity of the latus rectum.

(d) Prove that the curvature at any point of a circle of radius r is constant and equals the reciprocal of its radius.

(e) State Gauss Divergence Theorem.

(2 × 5 = 10)

Part A

Question II Determine the convergence of the following series. Give reasons for your answer:

(i) $\sum_{n=1}^{\infty} \frac{\sin^2 n}{2^n}$ (ii) $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^{1.1}}$ (iii) $\sum_{n=1}^{\infty} a_n$ where $a_n = \begin{cases} n/2^n, & n \text{ odd} \\ 1/2^n, & n \text{ even.} \end{cases}$

(iv) $\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$ (v) $\sum_{n=1}^{\infty} \frac{n!}{10^n}$

(10)

Question III (a) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

(b) Let $f(x, y)$ be a function of 2 variables x, y having continuous first order partial derivatives. Let $x = r\cos\theta$, $y = r\sin\theta$. Using the chain rule

$$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} \quad \text{and} \quad \frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial \theta},$$

find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ in terms of r, θ, $\frac{\partial f}{\partial r}$, $\frac{\partial f}{\partial \theta}$ and hence convert the equation $\frac{\partial f}{\partial x} + x\frac{\partial f}{\partial y} = y$ to the polar coordinates (r, θ).

(5+5=10)

Question IV (a) Find the extreme values which the function $f(x, y) = xy$ takes on the ellipse $\frac{x^2}{8} + \frac{y^2}{2} = 1$

(b) Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

(5+5=10)

P.T.O.

Part B

Question V (a) Evaluate the double integral $\int\int_R f(x,y)dA$ where $f(x,y) = 1 - 6x^2y$ and $R: 0 \leq x \leq 2, \ -1 \leq y \leq 1$.

(b) The region in the first quadrant enclosed by the parabola $y = x^2$, the y-axis, and the line $y = 1$ is revolved about the line $y = 3/2$ to generate a solid. Find the volume of the solid.

(5+5=10)

Question VI (a) Define curvature for a smooth differentiable curve $\vec{r}(t)$ in space. Give a mathematical formula for curvature and hence find the curvature of the helix $\vec{r}(t) = 2\cos t\,\hat{i} + 2\sin t\,\hat{j} + t\,\hat{k}$.

(b) Show that the field $\vec{F} = y\,\hat{i} + x\,\hat{j} + 4\,\hat{k}$ is exact and hence evaluate the integral $\int_{(1,1,1)}^{(2,3,-1)} y\,dx + x\,dy + 4\,dz$ over any curve joining the points $(1,1,1)$ and $(2,3,-1)$ by finding a potential function for $\vec{F}$.

(5+5=10)

Question VII (a) Without actually calculating the tangent and normal vector $\vec{T}$ and $\vec{N}$, write the acceleration $\vec{a}$ in the form $\vec{a} = a_T\vec{T} + a_N\vec{N}$ for the motion $\vec{r}(t) = \ln(t^2+1)\,\hat{i} + (t - 2\tan^{-1} t)\,\hat{j}$.

(b) State the circulation-curl form of the Green's theorem and use it to evaluate the line integral $\oint_C (3y\,dx + 2x\,dy)$ where C is the boundary of the curve $0 \leq x \leq \pi, \ 0 \leq y \leq \sin x$ in the positive sense.

(5+5=10)

x-x-x