Exam.Code:0907
Sub. Code: 6295

2122

B.E. (Bio-Technology) Third Semester

MATHS-302: Linear Algebra and Operations Research

(Common with IT)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.*

x-x-x

1. (a) Define linear combination, linearly dependence and independence of vectors. For what value of k the vector $u = (1, -2, k)$ can be expressed as a linear combination of vectors: $v = (3, 0, -2)$ and $w = (2, -1, -5)$ in $R^3(R)$.

(b) Define similar matrices. Prove that similar matrices have the same eigenvalues.

(c) Examine the convexity of the set $S = \{(x, y): x^2 + y^2 \leq 1\}$. Justify your answer.

(d) Write the dual of the following linear programming problem:

$$\text{Max } z = 5x_1 + 12x_2 + 4x_3$$

s. t. $x_1 - 3x_2 + x_3 \geq 4,\ 2x_1 - x_2 - 3x_3 = 8,\ x_1, x_2, x_3 \geq 0.$

(e) Write the differences and similarities between assignment and transportation problems.

SECTION-A

2. (a) Prove that the set $\{x^3 - x + 1, x^3 + 2x + 1, x + 1\}$ is linearly independent set of vectors in the vector space of all polynomials over the field of real numbers.

(b) Using Gauss-Jordan method, find the inverse of a matrix: $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$.

(c) Find the eigenvalues and the corresponding eigenvectors of the matrix:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

3. (a) Examine whether $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 2 & 0 \end{bmatrix}$ is diagonalizable or not? If yes, obtain the matrix P such that $P^{-1}AP$ is a diagonalizable.

(b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then prove that $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$. Hence, find A^{30}.

P.T.O.

4. (a) Find all basic feasible solutions for the problem: Max. $z = x_1 + 2x_2$ such that

$$x_1 + x_2 \leq 10,\ 2x_1 - x_2 \leq 40,\ x_1, x_2 \geq 0.$$

(b) Solve the LPP using the big M-method: Minimize $z = 2x_1 + 8x_2 + x_3$ such that

$$x_1 + x_2 \geq 2,\ 2x_2 + x_3 \geq 5,\ x_1, x_2, x_3 \geq 0.$$

SECTION-B

5. (a) Write down the dual of the LPP and solve it: Max. $z = 4x_1 + 2x_2$ such that $-x_1 - x_2 \leq -3,\ -x_1 + x_2 \leq -2$ and $x_1, x_2 \geq 0$. Hence or otherwise write down the solution of the primal.

(b) Solve by dual simplex method: Min. $z = 2x_1 + x_2$ subject to the constraints

$$3x_1 + x_2 \geq 3,\ 4x_1 + 3x_2 \geq 6,\ x_1 + 2x_2 \geq 3,\ x_1, x_2 \geq 0.$$

6. (a) Find the optimal solution of the following transportation problem:

	D_1	D_2	D_3	D_4	D_5	Availability
O_1	1	2	6	2	3	800
O_2	3	4	5	8	1	600
O_3	3	1	1	2	6	200
O_4	4	7	3	5	4	400
Requirement	400	100	700	300	500	

(b) Define assignment problem along with its areas of applications. Explain the difference between an assignment problem and a transportation problem.

7. A project has the following time schedule:

Activity	Time in weeks	Activity	Time in weeks
(1-2)	4	5-7	8
(1-3)	1	6-8	1
(2-4)	1	7-8	2
(3-4)	1	8-9	1
(3-5)	6	8-10	8
(4-9)	5	9-10	7
(5-6)	4		

Construct PERT network and compute: (i) T_E and T_E , (ii) Float of each activity, (iii) Critical path and its duration.

x-x-x