Exam. Code: 0927
Sub. Code: 6571

2122

B.E. (Electronics and Communication Engineering)

Third Semester

MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section. All questions carry equal marks.*

x-x-x

1 (a) What is the difference between linearly independent and dependent vectors. Prove that the vectors $u = (1, 2, -3)$, $v = (1, -3, 2)$ and $w = (2, -1, 5)$ of $\mathbb{R}^3(\mathbb{R})$ is linearly independent.

(b) Define linear transformation. Find a linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$.

(c) If λ is an eigen value of square matrix A, then prove that λ^2 is an eigen value of A^2.

(d) Can the residue at a singularity be zero? Can the residue at a simple pole be zero? Justify.

(e) What is a linear fractional transformation? What can you do with it? List special cases.

$(5 \times 2 = 10)$

Section –A

2 (a) Show that the set $V = \{a + b\sqrt{2} + c\sqrt[3]{3} : a, b, c \in \mathbb{Q}\}$ form a vector space over the field $\mathbb{Q}$ under usual addition and multiplication of real numbers.

(b) Find a basis and dimension of the solution space

$$\begin{cases} x + 2y + 4z + s = 0, \\ 2x + y + 5z - 2s + 2t = 0, \\ x - y + z + s + 6t = 0. \end{cases}$$

3 (a) Let the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined as $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Verify Rank – Nullity theorem for T.

(b) If $B_1 = \{(1, 0), (0, 1)\}$ and $B_2 = \{(1, 2), (2, 3)\}$ are basis of $\mathbb{R}^2$, then find the transition matrices P and Q from basis B_1 to B_2 and B_2 to B_1 respectively. Also verify that $Q = P^{-1}$.

4 (a) Prove that a linear transformation $T: V \to W$ is non-singular if and only if set of images of a linearly independent set is linearly independent.

(b) State Cayley–Hamilton theorem along with its applications. Verify the same for the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 2 & -1 & 0 \end{bmatrix}$. Hence find A^{-1}.

Section–B

5 (a) If $f(z)$ is differentiable at z_0, then show that $f(z)$ is continuous at z_0.

(b) Find u and v so that $f(z) = \frac{z-i}{z+i}$ is of form $f(z) = u(x,y) + iv(x,y)$. Determine all points (if any) at which the Cauchy-Riemann equations are satisfied and determine all points at which the function is differentiable.

(c) Solve: $\sin z = 100$.

6 (a) Find the Laurent series expansion for $f(z) = \frac{7z-2}{z^3-z^2-2z}$ in the regions given by

(i) $1 < |z+1| < 3$, (ii) $|z+1| > 3$.

(b) Evaluate the integral $I = \int_0^{2\pi} \frac{1+\sin\theta}{3+\cos\theta}\, d\theta$.

7(a) Show that the mapping $w = 1/z$ maps every straight line to a circle or a straight line, and every circle to a circle or straight line. Give an example of a circle that maps to a line, and a line that maps to a circle.

(b) Write a note on different types of singularities with suitable examples. Find all the singularities of the function $f(z) = e^{1/(1-z)}$ in the finite plane and the corresponding residues.

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