2122
B.E. (Computer Science and Engineering)
Third Semester
CS-303: Discrete Structures

Time allowed: 3 Hours   **Max. Marks: 50**

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section.*

x-x-x

1. Briefly explain the following with example:

   (a) Asymmetric and Anti-symmetric relations

   (b) Bipartite graph

   (c) Group

   (d) Well ordered Poset

   (e) Predicates   **(5x2=10)**

Section-A

2. Prove the validity of following arguments without using truth tables.

   (a) $(p \wedge q) \to r, p \to q \vdash p \to ((p \wedge q) \wedge r)$

   (b) $(p \wedge q) \to r, (r \to q), (r \to q) \to (q \wedge r) \vdash (p \wedge q) \to (q \wedge r)$

   (c) $(p \vee q) \to \neg r, r \vee t, p \vdash t$

   (d) $p, q, (p \wedge q) \to r \vdash r$

   (e) $p \vee (q \to p), \neg p \wedge r \vdash \neg q$ **(5x2=10)**

3. (a) Consider A=B=C=R and let f: A→B and g: B→C be defined by f(x) = x+9 and $g(y) = y^2+3$. Find the following composition functions: (i) (fof)(a) (ii) (gof)(b) (iii) (gof)(4)   (iv) (fog)(-4).   **(4)**

   (b) Let A be the set of people and R be a relation on A defined by R = {(a, b): a is a brother of b}. Is R an Equivalence Relation or a Partial Order Relation? Justify.   **(3)**

   (c) Show that $\forall x P(x) \vee \forall x\, Q(x) \Rightarrow \forall x (P(x) \vee Q(x))$.   **(3)**

4. (a) Consider A={1,2,3,4,5,6,7,8,9} and let B_1={5,6,7}, B_2={2,4,5,9}, B_3={3,4,5,6,8,9}.

   I. Find the minsets generated by B_1, B_2, B_3.

   II. Do these minsets form a partition of A?   **(4)**

   (b) Check the validity of the following argument. "If I go to school, then I attend all classes. If I attend all classes, then I get A grade. I do not get grade A and I do not feel happy. Therefore, if I do not go to school then I do not feel happy."   **(3)**

   (c) Prove that for all non-negative real numbers x, y and z if $x^2 + y^2 = z^2$, then x+y>=z using a suitable method of proof.   **(3)**

P.T.O.

Section-B

5. (a) Define order and degree of a recurrence relation. Solve the recurrence relation $a_r - 6a_{r-1} + 9a_{r-2} = (r+1)3^r$. **(8)**

   (b) Is E, the set of even integers a commutative ring without unity? Justify your answer. **(2)**

6. (a) Solve the recurrence relation $S(k) - 3S(k-1) - 2 = 0$, $k \geq 1$, where $S(0) = 1$, by using generating functions. **(5)**

   (b) Find the shortest path and its length from P to Q by using Dijkstra's algorithm in the following graph. **(5)**
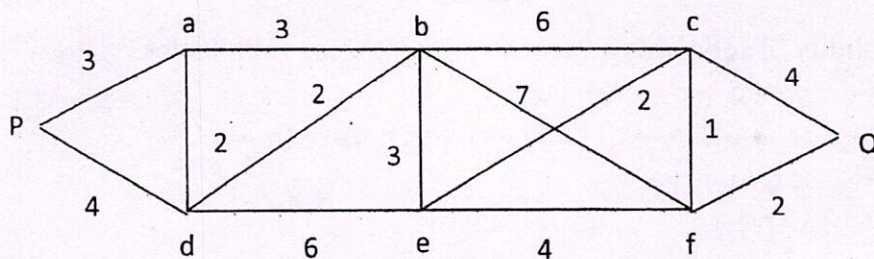
7. (a) Find the number of ways in which a registration number of 6 digits (repetition of digits is not allowed) can be formed using the set of letters in the English alphabet and the numbers 0,1,....,9, if the following conditions are to be satisfied: **(5)**

   I. The registration number starts with 2 letters followed by 4 numbers.

   II. The registration number starts with UK followed by 2 letters and then 2 numbers.

   (b) Which of the following are groups under addition N, Z, Q, R, C? Prove. **(5)**

x-x-x