Exam.Code:0933
Sub. Code: 6660

2122

B.E. (Electrical and Electronics Engineering)

Third Semester

BS-EE-305: MATH-III

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section.*

x-x-x

1 (a) Define linear span of a vector space. What is the linear span of the set $S = \left\{e_1, e_2, \left(\frac{e_1+e_2}{2}\right)\right\}$, where e_1 and e_2 are unit vectors?

(b) Show that a linear mapping $f: V \rightarrow U$ is one-one if and only if $\ker(f) = 0$.

(c) Prove that similar matrices have same eigenvalues.

(d) Define analytic and non-analytic function with suitable examples. Examine analyticity of $f(z) = \ln|z| + i\,\mathrm{Arg}(z), z \neq 0$.

(e) Define bi-linear transformation. Find its fixed and critical points. $(5\times 2 = 10)$

SECTION-A

2. (a) Find the column rank of the matrix: $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}$.

(b) Let V be the vector space of all square matrices over R. Determine which of the following are sub-spaces of V?

(i) $W = \left\{\begin{bmatrix} x & y \\ z & 0 \end{bmatrix} : x, y, z \in R\right\}$ (ii) $W = \left\{\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} : x, y \in R\right\}$. Justify.

(c) Test the consistent of the linear system: $x + 2y + z = 3$; $2x + y + 3z = 5$; $2x + 4y + 2z = 7$.

3. (a) Prove that the set of all positive real numbers with operations: $x + y = xy$ and $k\,x = x^k$ is a real vector space.

(b) Extend the set of vectors (1, 2, 3), (2, 1, 0) to form a basis of R^3.

P.T.O.

(c) Prove that linear transformation $T: R^3 \rightarrow R^2$ defined by $T(x, y, z) = (x, y)$ is a linear transformation and is onto but not 1-1.

4. (a) Examine whether $A = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ is diagonalizable or not? If yes, obtain the matrix P such that $P^{-1}AP$ is a diagonalizable.

(b) Find the matrix representing the transformation $T: R^3 \rightarrow R^4$ defined by $T(x,y,z) = (x+y+z,\ 2x+z, 2y-z,\ 6y)$ relative to the standard basis of R^3 and R^4.

SECTION-B

5. (a) Solve: (i) $\sin\left(\frac{i}{z}\right) = i$, (ii) $\sinh z = i$.

(b) Prove that $w = \cos z$ is not a bounded function.

(c) Find all bilinear transformations whose fixed points are i and $-i$.

6. (a) State Laurent's theorem. Find all possible Taylor's and Laurent series expansions for the function $f(z) = \frac{1}{1-z}$ about $z = 0$.

(b) State Cauchy residue theorem. Use it to evaluate $\int_C \frac{4-3z}{z(z-1)(z-2)} dz$, where c is the circle $|z| = \frac{3}{2}$.

7. (a) Explain different types of singularities with suitable examples. Find all singularities of the function $w = \frac{\sin z}{\sinh z}$ and classify them.

(b) Evaluate the integral using contour integration: $I = \int_0^{2\pi} \frac{\sin\theta}{3+\cos\theta} d\theta$.

x-x-x