Exam.Code:0921
Sub. Code: 6471

2122

B. E. (Information Technology)

Third Semester

ASM-301: Linear Algebra and Probability Theory

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. 1 (Section-A) which is compulsory and selecting two questions each from Section B-C.*

x-x-x

Section - A

1. Answer the following:

a) Define vector space and subspace.

b) Define Kernel of a Linear mapping $f : V \to U$. Show that Kernel of f is a subspace of V.

c) Let A and B be events with $P(A) = \frac{3}{8}$, $P(B) = \frac{5}{8}$ and $P(A \cup B) = \frac{3}{4}$. Find $P(A|B)$.

d) Let the density of X be $f(x) = \begin{cases} 1/2 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$. Find the density of $3X - 2$.

e) A bag contains 1 red and 7 white marbles. A marble is drawn from the bag, and its color is observed. Then the marble is put back into the bag and the contents are thoroughly mixed. Find the probability that in 5 such drawings, a red ball is selected exactly 3 times. $(5\times 2 = 10)$

Section - B

2. a) Let W be the subspace of $\mathbb{R}^5$ spanned by $u_1 = (1, 2, -1, 3, 4)$, $u_2 = (2, 4, -2, 6, 8)$, $u_3 = (1, 3, 2, 2, 6)$, $u_4 = (1, 4, 5, 1, 8)$, $u_5 = (2, 7, 3, 3, 9)$. Find a subset of the vectors that form a basis of W.

b) Consider the subspace $U = \{(a, b, c, d) : b - 2c + d = 0\}$ and $W = \{(a, b, c, d) : a = d, b = 2c$ of $\mathbb{R}^4$. Find the basis and dimension of $U \cap W$.

c) Find the basis and dimension of the subspace W of $\mathbb{R}^3$ where: (i) $W = \{(a, b, c) : a + b + c = 0\}$, (ii) $W = \{(a, b, c) : a = b = c\}$. $(03 + 04 + 03)$

3. a) Find all the eigenvalues and the corresponding eigenvectors of the matrix $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$. Check whether A is diagonalizable or not? If yes, find P such that $D = P^{-1}AP$ is diagonal.

b) Let G be the linear operator on $\mathbb{R}^3$ defined by $G(x, y, z) = (2y + z,\ x - 4y,\ 3x)$. Find the matrix representation of G relative to the basis $S = \{(1, 1, 1),\ (1, 1, 0),\ (1, 0, 0)\}$. Verify that $[G][v] = [G(v)]$ for any vector v in $\mathbb{R}^3$. $(05 + 05)$

4. a) Consider the system of linear equations $x + 2y + z = 3$, $ay + 5z = 10$, $2x + 7y + az = b$. Find the values of a for which the system has a unique solution.

b) Find the dimension and a basis of the general solution W of each of the system of linear equations $2x_1 - 4x_2 + 3x_3 - x_4 + 2x_5 = 0$ and $3x_1 - 6x_2 + 5x_3 - 2x_4 + 4x_5 = 0$ $5x_1 - 10x_2 + 7x_3 - 3x_4 + 4x_5 = 0$.

c) Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear mapping for which $F(1, 2) = (2, 3)$ and $F(0, 1) = (1, 4)$.

$(04 + 03 + 03)$

P.T.O.

Section - C

5. a) A rifleman hits target with probability .4. He fires four times. Determine the elements of the event A that the man hits the target exactly twice: and find $P(A)$. Also, find the probability that the man hits the target at least once.

b) An urn contains 5 red marbles and 3 white marbles. A marble is selected at random from the urn, discarded, and two marbles of the other color are put into the urn. A second marble is then selected from the urn. Find the probability that (i) the second marble is red, (ii) both marbles are of the same color.

c) Let X be a random variable with the distribution given in Table 1 and let $Y = X^2$.

x_i	-2	-1	1	2
$f(x_i)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Table 1:

Determine (i) the distribution g of Y, (ii) the joint distribution h of X and Y, (iii) covariance(X, Y) and correlation(X, Y) (04 + 04 + 02)

6. a) Let X and Y have joint density function $f(x,y) = \begin{cases} x+y & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$. Find the conditional expectation of (a) Y given X, (b) X given Y.

b) Find a least-square line to the data given in Table 2 using (i) x as the independent variable, (ii) x as the dependent variable.

x	3	5	6	8	9	11
y	2	3	4	6	5	8

Table 2:

Further find the value of y when $x = 12$ and value of x when $y = 7$.

c) A random variable X has density function given by

$$f(x,y) = \begin{cases} 2e^{-2x} & x \ge 0 \\ 0 & x < 0 \end{cases}$$

Find (a) the moment generating function, (b) the first four moments about the origin

(04 + 03 + 03)

7. a) Show that if p is small and n is large, then the binomial distribution is approximated by the Poisson distribution; that is, $b(k; n, p) \equiv p(k; \lambda)$ where $\lambda = np$.

b) Suppose the weights of 2000 male students are normally distributed with mean 155 pounds and standard deviation 20 pounds. Find the number of students with weights (i) less than or equal to 100 pounds, (ii) between 120 and 130 pounds, (iii) between 150 and 175 pounds, (iv) greater than or equal to 200 pounds. (05 + 05)

x-x-x