Printed Pages: 4 Roll No. ....

(ii) Questions : 7

Sub. Code : 6 1 9 3

Exam. Code : 0 9 0 5

B.Engg. 1st Year (1st Semester)

(2122)

BIO-TECHNOLOGY (Calculas)

(Common to All Streams)

Paper : ASM-101

Time Allowed : Three Hours] [Maximum Marks : 50

Note :—Attempt **five** questions in all including Question No. **1** which is compulsory and selecting **two** questions from each Section.

SECTION—A

1. Answer the following :

(a) Let $\begin{cases} \dfrac{\sin(x-y)}{|x|+|y|}, & |x|+|y| \neq 0 \\ 0, & (x, y) = (0, 0) \end{cases}$. Check for the continuity of the function f at the origin ?

(b) If $z = x + f(u)$ where $u = xy$, show that $x\dfrac{\partial z}{\partial x} - y\dfrac{\partial z}{\partial y} = x$.

(c) Find the area of the surface generated by revolving the curve $y = 2\sqrt{x}, 1 \leq x \leq 2$, about the x-axis.

(d) A projectile is fired from the origin over horizontal ground at an initial speed of 300 m/sec and a launch angle of 60°. Where will the projectile be 5 sec later ?

(e) State Gauss divergence theorem. $5\times2=10$

SECTION—B

2. (a) For approximately what values of x can you replace $\sin x$ by $x - \frac{x^3}{6}$ with an error of magnitude no greater than 5×10^{-4} ?

(b) Determine the number of terms should be used to estimate the sum of the series $\sum(-1)^{n+1}\frac{1}{(n+3\sqrt{n})^3}$ with an error of less than 0.01.

(c) Check the convergence of the series :

(i) $\sum\frac{1}{\sqrt{n}(\sqrt{n}+1)}$

(ii) $\sum\frac{(\ln n)^2}{n^{3/2}}$. 3+4+3

3. (a) For what values of x does the series $\sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{\sqrt{n+3}}$ converge absolutely and conditionally.

(b) Find the linearization L(x, y, z) of the function $f(x, y, z) = xy + 2yz - 3xz$ at $P_0(1, 1, 0)$. Also find the upper bound for the magnitude of the error E in the approximation $f(x, y, z) \approx L(x, y, z)$ over the region $R : |x - 1| \leq 0.01, |y - 1| \leq 0.01, z \leq 0.01$.

(c) If f(u, v, w) is differentiable and u = x − y, v = y − z and w = z − x, show that $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$. 3+4+3

4. (a) Find the dimensions of the rectangular box of maximum volume that can be inscribe inside the ellipsoid $x^2 + y^2 + z^2 = 4$.

(b) The plane x + y + z = 1 cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farther from the origin.

(c) Use Taylor's formula to find a quadratic approximation of $e^x \sin y$ at the origin. Estimate the error in the approximation if $|x| \le 0.1$ and $|y| \le 0.1$. 4+3+3

SECTION—C

5. (a) Use Stoke's theorem to evaluate

$$\int_C \tilde{F}\cdot d\bar{r} \text{ if } \tilde{F} = xy\hat{i} + xy\hat{j} + 3xz\,\hat{k}$$

where C is the boundary of the portion of the plane 2xy + y + z = 2 in the first octant traversed counterclockwise.

(b) Show that the vector field

$$\int_C \tilde{F} = (y\sin z)\hat{i} + (x\sin z)\hat{j} + (xy\cos z)\hat{k}$$

is conservative over its natural domain and find a potential function for it. 5+5

6. (a) Find the volume of the solid in the first octant bounded by the co-ordinate planes, the cylinder $x^2 + y^2 = 4$ and the plane $z + y = 3$.

(b) Evaluate the integral $\int_0^\infty \int_0^\infty \frac{1}{(1 + x^2 + y^2)^2} dxdy$.

(c) Find the moment of inertia of the ice cream cone cut from the solid sphere $\rho \leq 1$ by the cone $\phi = \frac{\pi}{3}$, whereas the constant density of the solid is 1. 3+3+4

7. (a) Find the length of the curve

$$\vec{r}(t) = (\sqrt{2t})\hat{i} + (\sqrt{2t})\hat{j} + (1 - t^2)\hat{k}$$

from $(0, 0, 1)$ to $(\sqrt{2}, \sqrt{2}, 0)$.

(b) Find the curvature (κ), unit tangent vector (T), principal Unit Normal Vector (N) and binormal vector (B) of the helix :

$$\vec{r}(t) = (a \cos t)\hat{i} + (a \sin t)\hat{j} + bt\hat{k},\ a, b \geq 0,\ a^2 + b^2 \neq 0$$

Also find the largest value of torsion (τ) of the helix for given value a.

(c) Let $f(x, y) = x^2 - xy + y^2 - y$. Find the directions u and directional derivative $D_u f(1, -1)$ for which $D_u f(1, -1) = -3$. 2+6+2