Exam.Code:0921
Sub. Code: 6831

2021

B. E. (Information Technology)

Third Semester

MATH-303: Linear Algebra and Probability Theory

Time allowed: 3 Hours | Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.*

x-x-x

Question I (a) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

(b) When is a system of linear equations said to be consistent? What is the condition on the system in terms of rank of the matrices involved for it to be consistent.

(c) Express $v = (3, 7, -4)$ in $\mathbb{R}^3$ as a linear combination of the vectors $u_1 = (1, 2, 3)$, $u_2 = (2, 3, 7)$, $u_3 = (3, 5, 6)$.

(d) Let X be a random variable with mean μ and standard deviation σ, then find the mean and variance of the standardized random variable $X^* = \frac{X-\mu}{\sigma}$.

(e) If X and Y are two independent random variables, show that $E(XY) = E(X)E(Y)$.

(2 × 5 = 10)

Part A

Question II (a) Solve the following system of linear equations using Gauss elimination method.

$$7x_1 + 2x_2 - 2x_3 - 4x_4 + 3x_5 = 8$$
$$-3x_1 - 3x_2 + 2x_4 + x_5 = -1$$
$$4x_1 - x_2 - 8x_3 + 20x_5 = 1$$

(b) State the Cayley-Hamilton theorem. Using it, invert the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}.$$

(5+5=10)

Question III (a) Find the inverse of the following matrix using Gauss-Jordan elimination method:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

(b) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 2 & 0 & -2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

Question IV (a) Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear mapping defined by $F(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$. Find a basis and the dimension of the image of F and the kernel of F.

(b) Consider the following two basis of $\mathbb{R}^3$:
$E = \{e_1, e_2, e_3\} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$
$S = \{u_1, u_2, u_3\} = \{(1, 0, 1), (2, 1, 2), (1, 2, 2)\}$.
Find the change of basis matrix Q from the basis S to basis E.

(5+5=10)

P.T.O.

Part B

Question V (a) A box contains 9 tickets numbered from 1 to 9, inclusive. If three tickets are drawn from the box 1 at a time, find the probability that they are alternately either odd , even, odd or even, odd, even.

(b) In a bolt factory, Machines A, B and C manufacture respectively 25 %, 35 % and 40 % of the total. Of their output 5, 4 2 percents are defective bolts. A bolt is drwan at random from the product and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C?

(4+6=10)

Question VI (a) Let the random variable X assume value 'n' with the probability law:

$$P(X = n) = pq^{n-1}; n = 1, 2, 3, \ldots$$

Find the moment generating function and hence mean and variance.

(b) Two random variables X and Y have joint probability density function as:

$$f(x, y) = \begin{cases} 2 - x - y; & 0 \leq x \leq 1, 0 \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) Marginal probability density function of X and Y.
(ii) Variance of X and Y
(iii) Covariance between X and Y.

(4+6=10)

Question VII (a) An insurance sales representative sells policies to 5 men, all of identical age and in good health. According to the acturial table, the probability that a man of this particular age will be alive 30 years hence is 2/3. Find the probability that in 30 years (i) all 5 men, (b) at least 3 men, (iii) only 2 men, (iv) at least 1 man will be alive.

(b) If the diameters of ball bearings are normally distributed with mean 0.6140 inches and standard deviation 0.0025 inches, determine the percentage of ball bearing wih diameters
(i) between 0.610 and 0.618
(ii) greater than 0.635
(iii) Equal to 0.667

(c) If 3% of the electric bulbs manufactured by a company are defective, using Poisson distribution find the probability that in a sample of 100 bulbs,

(i) 0 (ii) 2 (iii) 5 bulbs will be defective.

(4+3+3=10)

x-x-x