Exam.Code:0933
Sub. Code: 6974

2021
B.E. (Electrical and Electronics Engineering)
Third Semester
MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours Max. Marks: 50

***NOTE**: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.*

x-x-x

Question I (a) Find the rank of the following matrix by reducing it to its row echelon form:

$$A = \begin{bmatrix} 1 & -2 & 0 \\ 3 & 2 & 1 \\ 2 & 3 & 7 \\ -1 & 2 & 0 \end{bmatrix}$$

(b) Let $P_2(t)$ be the vector spaces of all polynomials of degree ≤ 2 in a single variable t. Show that the polynomials $p_1 = t+1$, $p_2 = t-1$ and $p_3 = (t-1)^2$ form a basis of $P_2(t)$.

(c) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

(d) Solve the equation $e^{2z-1} = 2$ for complex number z.

(e) State Cauchy's Residue theorem. Use it to evaluate $\int_{|z|=1} \frac{1}{z} dz$

(2 x 5 = 10)

Part A

Question II (a) Solve the following system of linear equations using Gauss elimination method.

$$2x_1 + \frac{1}{2}x_2 + 3x_3 + \frac{1}{3}x_4 = 1$$

$$\frac{1}{5}x_1 + 2x_2 + x_3 + 5x_4 = 0$$

$$x_1 + x_2 + x_3 + 2x_4 = 2$$

(b) When are two square matrices of the same order said to be similar? Prove that two similar matrices have the same eigen values.

(5+5=10)

Question III (a) Find the eigen values and eigen vectors of the following matrix A.

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 2 & 0 & -2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Is this matrix diagonalizable?

(b) Let $G: \mathbf{R}^3 \to \mathbf{R}^3$ be the linear mapping defined by $G(x, y, z)) = (x + 2y - z, y + z, x + y - 2z)$ Find a basis and the dimension of (a) the image of G, (b) the kernel of G.

(5+5=10)

Question IV (a) State the Cayley-Hamilton theorem. Using it, invert the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}.$$

P.T.O.

(b) Consider the following two bases of $\mathbb{R}^2$:
$S = \{u_1, u_2\} = \{(1, -2), (3, -4)\}$ and $S' = \{v_1, v_2\} = \{(1, 3), (3, 8)\}$

(i) Find the change of basis matrix P from S to S'.
(ii) Find the change of basis matrix Q from S' to S.
(iii) Verify that $Q = P^{-1}$

(5+5=10)

Part B

Question V (a) (i) Find all roots of the equation $\sin z = \cosh 4$ by equating the real parts and the imaginary parts of $\sin z$ and $\cosh 4$.

(ii) Find all roots of the equation $\cos z = 2$.

(b) Let u and v denote the real and imaginary components of the function f defined by the equations

$$f(z) = \begin{cases} \dfrac{(\bar{z})^2}{z} & \text{when} \quad z \neq 0, \\ 0 & \text{when} \quad z = 0. \end{cases}$$

Verify that the Cauchy-Riemann equations are satisfied at the origin $z = 0$ but $f'(0)$ nevertheless fails to exist.

(5+5=10)

Question VI (a) Define the complex logarithm function. Discuss its continuity and differentiability.

(b) Use the Cauchy's residue theorem to evaluate the integral $\int_C \frac{5z-2}{z(z-1)}dz$ when C is the circle $|z| = 2$ described counter clockwise.

(5+5=10)

Question VII (a) Give two Laurent series expansions in powers of z for the function $\frac{1}{z^2(1-z)}$ and specify the regions in which those expansions are valid.

(b) What the image of first quadrant in the z-plane by the mapping $f(z) = z^2$ in the w-plane? Explain.

(5+5=10)

x-x-x