

2021
B.E.(Bio-Technology) Third Semester
MATHS-302: Linear Algebra and Operations Research
(Common with IT)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section. All questions carry equal marks.

x-x-x

I. Answer the following:-

1. (a) What is basic philosophy behind the Gaussian elimination method for solving a system of linear equations? State difference between the Gaussian elimination method and Gauss-Jordan method.
- (b) Define similar matrices. Prove that similar matrices have the same eigenvalues.
- (c) Define convex set. Prove that the set $S = \{(x_1, x_2): 2x_1 + 3x_2 = 7\} \subset R^2$ is a convex set.
- (d) What do you mean by primal and dual problems?
- (e) Explain critical path analysis in project management.

SECTION-A

2. (a) Find the rank of the matrix: $A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & -1 & 4 \\ -2 & 8 & 2 \end{bmatrix}$.
- (b) Determine whether the set of vectors: $S = \{(1, 1, 2), (1, , 5), (5, 3, 4)\}$ forms a basis of R^3 .
- (c) Solve the system: $2x + z = 3$; $x - y + z = 1$; $4x - 2y + 3z = 3$ by Gaussian elimination method.
3. (a) Find the eigenvalues and the corresponding eigenvectors of the matrix:
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$
- (b) State Cayley-Hamilton theorem and verify it for the matrix: $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Find A^{-1} and A^8 .
4. (a) Solve the LP problem: Max. $z = 3x_1 + 5x_2 + 4x_3$, subject to the constraints:

P.T.O.

(2)

$$2x_1 + 3x_2 \leq 8, 2x_2 + 5x_3 \leq 10, 3x_1 + 2x_2 + 4x_3 \leq 15, \text{ and } x_1, x_2, x_3 \geq 0.$$

- (b) Solve the problem: Minimize $z = x_1 + x_2$, subject to $2x_1 + x_2 \geq 4$, $x_1 + 7x_2 \geq 7$ and $x_1, x_2 \geq 0$.

SECTION-B

5. (a) Explain the significance of duality theory in linear programming. For a given LP problem state its dual and show that the dual of the dual is the primal.
- (b) Use dual simplex method to solve: Max. $z = -2x_1 - x_3$, subject to
- $$x_1 - x_2 - x_3 \geq 5, x_1 - 2x_2 + 4x_3 \geq 8, x_1, x_2, x_3 \geq 0.$$
6. (a) Give the mathematical formulation and difference between Transportation and Assignment problems.
- (b) A manufacturer has distribution centres at Delhi, Calcutta and Chennai. These centres have available 30, 50, and 70 units of product. His four retail outlets require the following number of units A, 30; B, 20; C, 60; D, 40. The transportation cost per unit in rupees each centre and outlet is given in the following table: Determine the minimum transportation cost.

Distribution centres	Retail Outlets			
	A	B	C	D
Delhi	10	7	3	6
Kolkata	1	6	7	3
Chennai	7	4	5	3

7. (a) Give the mathematical formulation of an assignment problem. How does it differ from a transportation problem.
- (b) A project has the following time schedule:

Activity	Time in months	Activity	Time in months
(1-2)	2	(4-6)	3
(1-3)	2	(5-8)	1
(1-4)	1	(6-9)	5
(2-5)	4	(7-8)	4
(3-6)	8	(8-9)	3
(3-7)	5		

(3)

Construct PERT network and compute: (i) Critical path and its duration. Also find the minimum number of cranes the project must have for its activities 2-5, 3-7, and 8-9 without delaying the project. Then, there is any change required in PERT network? If so, indicate the name.

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