Exam.Code:1014
Sub. Code: 7749

2031
M.E. (Mechanical Engineering)
First Semester
MME-101: Advanced Engineering Mathematics

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, selecting atleast two questions from each Part. Use of simple calculator is allowed.*

x-x-x

PART A

I. (a) Find power series solution in powers of x of the differential equation:

$$(1-x^2)y'' + 2y = 0, y(0) = 4, y'(0) = 5$$

by developing the recurrence relation between coefficients. (5)

(b) Use Frobenius method to solve the differential equation:

$$xy'' + 2y' + xy = 0.$$ (5)

II. (a) Solve the Legendre's differential equation:

$$(1-x^2)y'' - 2xy' + n(n+1)y = 0.$$ (5)

(b) State and prove Rodrigues's formula. (5)

III. (a) Derive a general expression of Bessel's function $J_v(x)$ for $v \geq 0$ and hence evaluate $J_{1/2}(x)$. (5)

(b) Derive orthogonality relations for Bessel's functions. (5)

IV. (a) Solve the differential equation:

$$xy'' + 2y' + \frac{1}{2}xy = 0.$$ (5)

(b) Write following differential equation as STURM-LIOUVILLE equation. Find the eigen values and the eigen functions:

$$y'' + \lambda y = 0, y(0) = 0, y'(1) = 0.$$ (5)

PART B

V. (a) Using Picard's method, find a solution of

$$\frac{dy}{dx} = 1 + xy, \ y(0) = 0.$$ (5)

(b) Apply Runge-Kutta's method of fourth order to find an approximate value of y for $x = 0.2$ in steps of 0.1, given that:

$$\frac{dy}{dx} = x + y^2, \ y(0) = 1. \quad (5)$$

VI. Solve $\nabla^2 u = 0$ under the conditions $u(0,y) = 0$, $u(4,y) = 12 + y$; $0 \leq y \leq 4$, $u(x,0) = 3x$, $u(x,4) = x^2$; $0 \leq x \leq 4$. (10)

VII. Evaluate the pivotal values of $u_{tt} = 4\,u_{xx}$. The boundary conditions are $u(0,t) = u(4,t) = 0$ and initial conditions are $u_t(x,0) = 0, u(x,0) = 4x - x^2$. (10)

VIII. Solve the boundary value problem $u_t = u_{xx}$ under the conditions $u(0,t) = u(1,t) = 0$ and $u(x,0) = \sin\pi x$. (10)

x-x-x