Exam.Code:0918
Sub. Code: 6793

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B.E. (Computer Science and Engineering)
Sixth Semester
CS-602: Linear Algebra and Probability Theory

allowed: 3 Hours

Max. Marks: 50

: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Section.

x-x-x

1. (a) Define subspace of a vector space with suitable example. Prove that intersection of two subspace is again a subspace. $(5\times 2=10)$

(b) Explain linearly independent and dependent vectors with examples. Explain how they related to the rank of a given matrix?

(c) $T:R^2\rightarrow R^2$ defined by $T(x,y)=(2x+y,x-2y)$. Is T one-one? If so, find T^{-1}.

(d) What is Chebyshev's inequality? What does it measure and tell us?

(e) What is the moment of a function? Write down the uses of moment generating function.

SECTION-A

2. (a) Let V be a vector space of real valued continuous function over R. Prove that the set of solutions of differential equation: $5\frac{d^2y}{dx^2}-7\frac{dy}{dx}+2y=0$ is a subspace of V. $(3+3+4)$

(b) Prove that the set $\{x^3-x+1,x^2+2x+1,x+1\}$ is linearly independent set of vectors in the vector space of all polynomial over the field of real numbers.

(c) Prove that the set $\{(2,1,4),(1,-1,2),(3,1,-2)\}$ forms a basis of R^3.

3. (a) Examine whether the matrix $A=\begin{bmatrix}3 & 2 & 1\\ 0 & 2 & 0\\ 1 & 2 & 3\end{bmatrix}$ is diagonalizable? If so, find the matrix P such that $P^{-1}AP$ is a diagonal matrix.

P.T.O.

(b) Let $T: R^3 \to R^3$ be the linear mapping defined by $T(x, y, z) = \{x+2y-z, y+z, x+y-2z\}$. Find a basis and the dimension of (i) the image of T, (ii) the kernel of T.

4. (a) Prove that linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = \{x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta\}$ is a vector space isomorphism.

(b) Suppose the $x-$ and $y-axes$ in the plane R^2 are rotated counterclockwise 45° so that the new $x'-$ and $y'-axes$ are along the line $y = x$ and the line $y = -x$, respectively. (i) Find the change of basis matrix P, (ii) Find the coordinates of the point $A(5, 6)$ under the given rotation..

SECTION-B

5. (a) State and prove Baye's theorem.

(b) The chance that a doctor will diagnose a disease correctly is 70%. The chances of death of patient after correct diagnosis is 35%, while after wrong diagnosis is 80%. If a patient dies after treatment, find the probability that he was diagnosed (i) Wrongly, (ii) Correctly.

6. (a) Discuss the properties of marginal and conditional distributions.

(b) Define normal distribution. The distribution of weekly wages of 500 workers in a factory is approximately normal with a mean and standard deviation Rs. 75 and Rs. 15, respectively. Find the number of workers who received weekly wages (i) More than Rs. 90, (ii) Less than Rs. 45.

7. (a) For a Poisson distribution, find first four moments about origin and hence, find first four central moments. $(4+3+3)$

(b) State central limit theorem for the following two cases: (i) Equal distribution, (ii) Unequal distribution.

(c) Let $X \& Y$ be independent random variable having density functions: $f_X(x) = \begin{cases} e^{-x}, x \geq 0 \\ 0, x < 0 \end{cases}$ and $f_Y(y) = \begin{cases} 2e^{-y}, y \geq 0 \\ 0, y < 0 \end{cases}$. Find the density function of their sum $X+Y$.

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