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Exam.Code:0922
Sub. Code: 7014

1059

B.E. (Information Technology) Fourth Semester
MATHS-403: Discrete Structures

...llowed: 3 Hours

Max. Marks: 50

...: *Attempt five questions in all, including Question No. 1 (Section-A) which is compulsory and selecting two questions each from Section B-C.*

x-x-x

Section - A

... Answer the following:

a) Check whether $(p \wedge (\neg p \vee q)) \wedge \neg q$ is tautology or contradiction.

b) Define Klein Four group.

c) Define total order set and Lexicographic order with examples.

d) Let $R = \{(1,2), (2,2), (3,4), (4,1)\}$ be any relation. Check whether R is symmetric, antisymmetric or transitive. Also, find symmetric, antisymmetric and transitive closures of R.

e) If $f(x) = \sqrt{x^2+1}$ and $g(x) = x^3 + 2$, calculate $f \circ g$ and $g \circ f$.

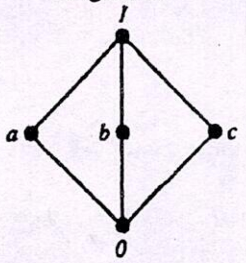
$(5 \times 2 = 10)$

Section - B

... a) Consider the set $A = R^2 - (0,0)$. Define a relation $\sim$ on A by $(x_1, x_2) \sim (y_1, y_2)$ if there exists $t > 0$ such that $x_1 = ty_1$, $x_2 = ty_2$. Prove that it is an equivalence relation. Describe equivalence class of (1, 2).

b) Define distributive Lattice. A Lattice is said to be modular if for all a, b, c, $a \preceq c$ implies that $a \vee (b \wedge c) = (a \vee b) \wedge c$. Show that a distributive lattice is modular. Show that the lattice shown in Figure 1 is non-distributive lattice that is modular.

Figure 1:



(05 + 05)

... a) Does the formula $f(x) = 1/(x^2 - 2)$ define a function $f : \mathbb{R} \to \mathbb{R}$ or a function $f : \mathbb{Z} \to \mathbb{R}$? Here $\mathbb{Z}$ and $\mathbb{R}$ stand for set of integers and set of real numbers, respectively.

P.T.O.

b) Suppose that in a group of 6 persons, each pair are either friends or enemies. Show that there are 3 persons who are either mutual friends or mutual enemies.

c) Let S be a set of six positive integers whose maximum is at most 14. Show that the sums of the elements in all the nonempty subsets of S cannot be all distinct.

(03+03+04)

4. a) Check the validity of the argument:
If there is a chance of rain or her red headband is missing, then Lois will not mow her lawn. Whenever the temperature is over 80°F, there is no chance for rain. Today the temperature is 85°F and Lois is wearing her red headband. Therefore Lois will mow her lawn.

b) Let $p(x), q(x)$ be open statements in the variable x, with a given universe. Prove that

$$\forall x\ p(x) \vee \forall x\ q(x) \Rightarrow \forall x\ [p(x) \vee q(x)]$$

Also find the counterexample for the converse.

(05 + 05)

Section - C

5. a) Define the following with suitable examples:
   i. Bipartiate Graph
   ii. Chromatic number of graph
   iii. Euler circuit and Hamiltonian circuit
   iv. Isomorphic graphs

b) In how many ways can the integers $1, 2, 3, \ldots, 10$ be arranged in a line so that no even integer is in its natural position?

(08 + 02)

6. a) Find and solve a recurrence relation for the number of binary sequences of length n that have no consecutive 0's.

b) Solve the following recurrence relation using method of generating functions:

$$a_{n+2} - 5a_{n+1} + 6a_n = 2,\ n \geq 0, \quad a_0 = 3, \quad a_1 = 7.$$

(05 + 05)

7. a) Define Cosets. Show that If G is a finite group of order n with H a subgroup of order m, then m divides n.

b) If G be a group with H finite and $H \subseteq G$, then H is a subgroup of G if and only if H is closed under the binary operation G.

(05 + 05)

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