

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.

x-x-x

- Q1. (a) What is the significance of asymptotic notation? [5 x 2]
(b) State the control abstraction for Divide and Conquer algorithm designing strategy.
(c) Compare greedy method and dynamic programming method.
(d) What is the Principle of Optimality?
(e) Explain the difference between explicit and implicit constraints using 8-queens problem.

Section – A

- Q2. (a) Solve the recurrence $T(n) = T(\sqrt{n}) + 1$ using substitution method. Your solution should be asymptotically tight. [5]
(b) Solve the recurrence $T(n) = 4T(n/2) + n^2 \lg n$ using master method. [5]
- Q3. (a) Write QuickSort algorithm based on divide and conquer strategy. Show how QuickSort sorts the array $A = [13, 19, 9, 5, 12, 8, 2, 6, 21]$. Also perform the space/time complexity analysis of QuickSort algorithm. [5]
(b) Write a short note on Strassen's Matrix Multiplication algorithm. How would you modify the same to multiply $n \times n$ matrices in which n is not an exact power of 2? Also perform the space/time complexity analysis of 1st algorithm as well as the modified algorithm. [5]
- Q4. (a) State the Greedy method based control abstraction for the subset paradigm. Apply it to solve the Knapsack problem. Find an optimal solution to the knapsack instance $n=7$, $m = 15$, $(p_1, p_2, \dots, p_7) = (10, 5, 15, 7, 6, 18, 3)$ and $(w_1, w_2, \dots, w_7) = (2, 3, 5, 7, 1, 4, 1)$. [5]
(b) Define a Spanning tree. Can Kruskal's algorithm return different spanning trees for the same input graph G ? If yes, justify the same. Also show that for each minimum spanning tree T of G , there is a way to sort the edges of G in Kruskal's algorithm so that the algorithm returns T . [5]

Section – B

- Q5. (a) Define the problem of Longest Common Subsequence. Also, design an algorithm to find the longest monotonically increasing subsequence of n numbers. Find the running time of the designed algorithm. [5]
(b) Write an algorithm AllPaths for All pairs shortest path problem using dynamic programming. Modify the function AllPaths so that a shortest path is output for each pair of vertices (i, j) . Also perform the space/time complexity analysis of the AllPaths algorithm as well as the modified algorithm. [5]

(2)

- Q6. (a) Write an algorithm NQueens for n-queens problem using backtracking. Further redefine Place(k,i) function so that it either returns the next legitimate column or an illegal value. Modify the complete n – queens solution to accommodate above altered strategy. Will this alteration make the previous algorithm more efficient or not? Justify your observation. [5]
- (b) Write the recursive backtracking algorithm SumOfSub for sum of subsets problem. Let $m = 35$, run SumOfSub on the data
1. $w = \{5, 7, 10, 12, 15, 18, 20\}$
 2. $w = \{20, 18, 15, 12, 10, 7, 5\}$
 3. $w = \{15, 7, 20, 5, 18, 10, 12\}$
- Are there any visible differences in the computing time? [5]
- Q7. (a) State your observation on the commonly believed relationship among P, NP, NP – Complete and NP – Hard problems. [4]
- (b) Obtain a nondeterministic algorithm of complexity $O(n)$ to determine whether there is a subset of n numbers $a_i, 1 \leq i \leq n$, that sums to m . [3]
- (c) Show that clique optimization problem reduces to clique decision problem. [3]