Exam. Code: 0921
Sub. Code: 6831

1079

B. Engg. (Information Technology)

3rd Semester

MATHS-303: Linear Algebra and Probability Theory

Time allowed: 3 Hours | Max. Marks: 50

NOTE: *Attempt five questions in all, including Q. No. 1 which is compulsory and selecting atleast two questions from each Unit.*

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I. Attempt the following: -

(a) Express the polynomial t^2+4t-3 in $P(t)$ as a linear combination of the polynomials $t^2-2t+5,\ \ 2t^2-3t,\ \ t+1$.

(b) For the map (linear) $F:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $F(x,y,z)=(x+2y-3z, 2x+5y-4z, x+4y+z)$, find Kernal F.

(c) Sixty percent of the employees of the XYZ Corporation are college graduates. Of these, ten percent are in sales. Of the employees who did not graduate from college, eighty percent are in sales. What is the probability that an employee selected at random in neither in sales nor a college graduate.

(d) The probability of a man hitting a target is $\frac{1}{4}$. How many times, mist he fire so that the probability of his hitting the target atleast once in greater then $2/3$? (2+3+3+2)

UNIT-I

II. (a) Show that the linear operator T on $\mathbb{R}^3$, where $T(x,y,z)=(x-3y-2z, y-4z, z)$ is non singular and find a formula for T^{-1}.

(b) Check whether the matrix $\begin{bmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ is diagonlizable or not? (5+5)

III. (a) Consider the subspaces $U=\{(a,b,c,d); b-2c+d=0\}$ and $W=\{(a,b,c,d); a=d, b=2c\}$ of $\mathbb{R}^4$. Find a basis and dimension of $U \cap W$.

(b) Let $f:\mathbb{R}^3 \to \mathbb{R}^2$ be the linear map defined by $f(x,y,z)=(3x+2y-4z, x-5y+3z)$. Find the matrix of f relative to the bases of $\mathbb{R}^3$ and $\mathbb{R}^2$: $\{(1,1,1),(1,1,0),(1,0,0)\}$ & $\{(1,3),(2,5)\}$ (5+5)

Contd....P/2

IV. (a) Find the eigen vectors for the matrix $A=\begin{bmatrix}4 & 1 & -1\\ 2 & 5 & -2\\ 1 & 1 & 2\end{bmatrix}$

(b) Solve the system of equations:
$x+2y-4z=-4,\quad 2x+5y-9z=-10,\quad 3x-2y+3z=11.$ (5+5)

UNIT-II

V. (a) Let X and Y be jointly distributed with p.d.f as $f_{XY}(x,y)=\begin{cases}\frac{1}{4}(1+xy) & ; \ |x|1, |y|<1\\ 0 & ; \ otherwise\end{cases}$. Show that X and Y are not independent but X^2 and Y^2 are independent.

(b) A sample of 100 items is taken at random from a batch known to contain 40% defectives. What is the probability that the sample contains (i) atleast 44 defectives (ii) exactly 44 defectives? (5+5)

VI. (a) If X is a Poisson Variate such that $P(X=2)=9P(X=4)+90(X=6)$. Find mean of X.

(b) Given the joint density function of X and Y as:
$f(x,y)=\begin{cases}\frac{1}{2}xe^{-y} & ; \ 0<x<2,\ y>0\\ 0 & ; \ elsewhere\end{cases}$

Find the distribution of X+Y. (5+5)

VII. (a) Twenty five books are placed at random on a shelf. Find the probability that a particular book shall be: (i) always together and (ii) never together.

(b) If X is the number scored in a throw of a fair die, show that the Chebychev's inequality gives $P\{|X-\mu|>2.5\}<0.47$, where μ is the mean of X, while actual probability is zero. (5+5)

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