

B. Engg. (Bio-Technology)-3rd Semester
MATHS-302: Linear Algebra and Operations Research
(Common with IT)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Q. No. 1 which is compulsory and selecting atleast two questions from each Part-A & B. Use of non-programmable calculator is allowed.

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1. (a) Check whether following set is convex or not: $D = \{(x, y) : x + y \leq 4, xy \leq 2, x, y \geq 0\}$.
(b) Write the dual of the following problem:

$$\min \quad 2x + 3y$$

subject to $3x + y - z \leq 4, x - y + z \geq 4, x \geq 0, y \leq 0$ and z is unrestricted.

- (c) Find the solution of the following transportation problem by matrix minima method:

	D_1	D_2	D_3	Availability
S_1	4	2	11	35
S_2	7	12	5	20
Requirement	15	28	12	

- (d) Find the eigen values of the matrix and one of the corresponding eigen vector:

$$\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}$$

- (e) Define CPM and PERT.

(5 × 2 = 10)

PART A

2. (a) Show that $v = (1, -2, 5)$ in R^3 can be written as a linear combination of the vectors (5)

$$u_1 = (1, 1, 1), \quad u_2 = (1, 2, 3), \quad u_3 = (2, -1, 1)$$

- (b) Find the inverse of the matrix using Gauss-Jordan elimination method: (5)

$$\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$$

3. (a) Find the eigen values and the corresponding eigen vectors of the following matrix: (5)

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$$

(2)

(b) Using simplex method, find an optimal solution of the following problem: (5)

$$\max \quad x + y$$

subject to

$$x + 2y \leq 6, \quad 2x + y \geq 16, \quad x, y \geq 0$$

4. (a) Diagonalize the following matrix, if possible: (5)

$$B = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$$

(b) Use two-phase simplex method to solve the LPP: (5)

$$\max \quad z = x + y$$

subject to

$$2x + y \geq 4, \quad x + 7y \geq 7, \quad x, y \geq 0$$

PART B

5. (a) State and prove strong duality theorem. (5)

(b) Using dual simplex method find an optimal solution of LPP: (5)

$$\min \quad Z = 3x + 4y$$

subject to

$$2x - 3y \geq 6, \quad 4x + 5y \geq 20, \quad x, y \geq 0$$

6. (a) Solve the following assignment problem to minimize the total cost of assignment: (5)

$$\begin{pmatrix} 9 & 12 & 11 & 20 \\ 13 & 8 & 9 & 10 \\ 13 & 12 & 9 & 22 \\ 11 & 15 & 8 & 16 \end{pmatrix}$$

(b) Solve the following transportation problem to minimize the transportation cost: (5)

	D_1	D_2	D_3	Available
O_1	10	13	4	35
O_2	6	11	5	22
O_3	18	8	9	28
Requirement	20	40	25	

The cell entries are unit transportation costs.

7. A project has the following time schedule. (10)

Activity :	1-2	1-3	1-4	2-5	3-6	3-7	4-6	5-8	6-9	7-8	8-9
Duration :	2	2	1	4	8	5	3	1	5	4	3

(i) Draw the network diagram of the project.

(ii) Find the critical path and project duration.