

Time allowed: 3 Hours

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.
x-x-x

Max. Marks: 50

1. (a) Describe the examples of electrical, mechanical, and biomedical systems with the differential equations.
- (b) Specify the Nyquist rate and Nyquist interval for the signal:
 $x(t) = \text{sinc}(150t) + \text{sinc}^2(150t)$
- (c) Find the inverse Fourier transform of the signal: $X(\omega) = \delta(\omega)$
- (d) Determine the transfer function and impulse response for the causal LTI system described by the difference equation $y(n) - \frac{1}{2}y(n-1) = x(n) + 2x(n-1)$
- (e) Describe the causality of a system in terms of its impulse response.

(5 × 2 = 10)

Section-A

2. Find whether the following systems are Memoryless, Time- Invariant, Linear, Causal, and Stable. Justify your answer. (10)
 - a. $y(t) = x(t-2) + x(2-t)$
 - b. $y(t) = \int_{-\infty}^{2t} x(\tau) d\tau$
 - c. $y[n] = x[n] \sum_{k=-\infty}^{\infty} \delta[n-2k]$
 - d. $y[n] = \log_{10}(|x[n]|)$
3. (a) The unit impulse response of an LTI system is $h(t) = e^{-t}u(t)$. Find the system's zero state response $y(t)$ if the input signal is $x(t) = e^{-2t}u(t-3)$. (5)

(b) Use the classical method to solve (5)

$$\frac{d^2y(t)}{dt^2} + 5 \frac{dy(t)}{dt} + 6y(t) = x(t)$$

for the initial conditions of $y(0^+) = -\frac{1}{2}$, $\frac{dy(0^+)}{dt} = \frac{1}{2}$, and the input of $x(t) = e^{-t}u(t)$

4. (a) Consider a stable LTI system characterized by the differential equation (5)

$$\frac{d^2y(t)}{dt^2} + 4 \frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

Find the frequency response $H(\omega)$ and the impulse response $h(t)$ of the system.
Find the response of this system if the input $x(t) = e^{-t}u(t)$?

P.T.O.

(2)

(b) Determine the Fourier series coefficients of the signal:

$$a. \quad x(n) = 1 + \sin \frac{2\pi n}{N} + 3 \cos \frac{2\pi n}{N} + \cos \left(\frac{4\pi n}{N} + \frac{\pi}{2} \right)$$

(5)

Section-B5. (a) Find the Fourier Transform of the sequence $y(n) = (-1)^n a^n u(n)$, $|a| < 1$

(b) Determine all possible signals having z-transform

(5)

(5)

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

6. (a) Determine the Laplace transform of the signal:

$$x(t) = (1 + 0.5 \sin(t)) \sin(1000t) u(t)$$

(5)

(b) Find the final value of the signal using Z-transform:

(5)

$$X(z) = \frac{2z^{-1}}{1 - 1.8z^{-1} + 0.8z^{-2}}$$

7. (a) Compute the Hilbert Transform of the signal $x(t) = \text{sinc}(2t)$

(5)

(b) Find the inverse Laplace Transform of the following:

(5)

$$X(s) = \frac{2s + 5}{(s + 2)(s + 3)}$$

x-x-x