Exam. Code: 0907
Sub. Code: 6699

1079

B. Engg. (Bio-Technology)-3rd Semester
AS-301: Engg. Mathematics-III
(Common)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: *Attempt five questions in all, including Q. No. 1 which is compulsory and selecting atleast two questions from each Part-A & B. Each question carries equal marks. Use of simple calculator is allowed.*

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1. (a) State the sandwich theorem for sequences. Apply it to prove that $\left\langle \frac{\cos n}{n} \right\rangle$ is a convergent sequence.

(b) State the nth term test for divergence. Examine whether the series $\sum_{n=1}^{\infty} \frac{n+1}{n}$ is convergent or divergent.

$(5\times 2)=10$

(c) State Cayley-Hamilton theorem. Write down its any two applications.

(d) Explain the differences and similarities between e^x and e^z.

(e) Define fixed and critical points of the mapping. Find the same for $w=\frac{3z-4}{z+\sin z}$.

PART-A

2. (a) Examine the convergence or divergence of the following sequences:

(i) $a_n = \tanh n$, (ii) $a_n = \tan^{-1} n$, (iii) $a_n = n-\sqrt{n^2-n}$, (iv) $a_n = \ln n - \ln(n+1)$

(b) Examine the convergence or divergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $(p \, is \, a \, real \, consatnt)$, (ii) $\sum_{n=0}^{\infty} \frac{2^n+5}{3^n}$.

3. (a) State and prove Leibnitz test for alternating series. Examine the convergence, absolute convergence and divergence of the series: $\sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right)$.

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(b) If the system of equations: $x+ay+az=0$; $bx+y+bz=0$; $cx+cy+z=0$, where a,b,c are non-zero and non-unity, has a nontrivial solution, then prove that $\frac{a}{1-a}+\frac{b}{1-b}+\frac{c}{1-c}=1$.

4. (a) Examine whether the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ is diagonalizable or not.

If diagonalizable, obtain the matrix P such that $P^{-1}AP$ is a diagonalizable matrix.

(b) Verify Cayley-Hamilton theorem for the matrix: $A=\begin{bmatrix}1 & 2 & 0\\ 1 & 2 & 1\\ -1 & 1 & 2\end{bmatrix}$.

PART-B

5. (a) Examine whether $w=\cos z$ is a bounded or unbounded function.

(b) Prove that the function $f(z)=\begin{cases}\frac{(\bar{z})^2}{z}, & z\neq 0\\ 0, & z=0\end{cases}$ is not differentiable at $z=0$, even though C-R equations are satisfied there on.

(c) Discuss the analyticity of $f(z)=|z|^2$.

6. (a) Explain and discuss the different type of isolated singularities. Give one example of each. Further, prove that if an analytic function $w=f(z)$ has a pole of order m at $z=z_0$, then $\frac{1}{f(z)}$ has a zero of order m at $z=z_0$.

(b) Discuss the mapping $w=\sinh z$.

7. (a) State residue theorem. Hence, evaluate $\oint_C \frac{1}{\sinh z}dz$, where C is the circle $|z|=4$ using it.

(b) Evaluate $I=\int_0^{2\pi}\frac{d\theta}{2+\sin\theta}$ using complex integration.

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