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Exam.Code:0906
Sub. Code: 6660

1079

B.E. (Biotechnology)
Second Semester
MATHS-201: Differential Equations and Transforms

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part. Use of non programmable calculator is allowed.*

x-x-x

1. (a) Find the general solution of the differential equation: $y' = x\tan(y-x)+1$.
   (b) Using power series method solve the differential equation: $y' = -2xy$.
   (c) Find the differential equation of all the planes having equal x and y intercepts.
   (d) Find the Laplace transform of $f(t) = t\sin 2t$.
   (e) Find a period function and find the fundamental period of the function $f(x) = \cos(2x)$. $(5 \times 2 = 10)$

PART A

2. (a) Find the general solution of the differential equation: (5)

$$(D^3 + 2D + 4)y = e^x \cos x$$

   (b) Find the general solution of the differential equation using method of variation of parameters: (5)

$$(D^2 + 4D + 4)y = 2\,\frac{e^{-2x}}{x^2}$$

3. (a) Find the general solution of the ordinary differential equation (5)

$$(D^2 - 1)y = x\sin x + e^{-x}\,(1 - x^2)$$

   (b) Using convolution theorem find the inverse Laplace transform of (5)

$$\frac{s}{(s^2 + a^2)^2}$$

4. (a) If $L[f(t)] = \bar{f}(s)$, then find the Laplace transform of $\frac{f(t)}{t}$, provided it exits. (5)
   (b) Using Laplace transforms, find the particular solution of the differential equation: (5)

$$y'' + y = u(t-1), \quad y(0) = 0, \quad y'(0) = 20$$

P.T.O.

PART B

5. (a) Prove that (5)

$$\int_0^\infty \frac{\sin w}{w} \cos wx \, dw = \begin{cases} \frac{\pi}{2} & \text{if } 0 \le x < 1 \\ \frac{\pi}{4} & \text{if } x = 1 \\ 0 & \text{if } x > 1 \end{cases}$$

(b) Find the complex Fourier series of the periodic function: $f(x) = x$, if $-\pi < x < \pi$, $p = 2\pi$. (5)

6. (a) Find the general solution of the partial differential equation (5)

$$x(x^2 + 3y^2)\, p + -y(3x^2 + y^2)\, q = 2z(y^2 - x^2).$$

(b) Let $f(x) = 1 - x^2$ $(-1 < x < 1)$ be a periodic function with period $p = 2$. Find the Fourier series for $f(x)$. (5)

7. (a) Find the solution of one-dimensional heat equation corresponging to a bar of length L such that the temperature $u(x,t)$ at the ends of the bar is kept zero and the initial temperature of the bar is given by $u(x,0) = f(x)$: (10)

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c^2 = \frac{K}{\sigma \rho}$$