Exam. Code: 0918
Sub. Code: 6793

1019

Bachelor of Engineering (Computer Science and Engg.)
6^{th} Semester
CS – 602: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

Note: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions each from Unit I - II. Use of simple calculator and statistical table is allowed.

0-0-0

I. Attempt the following questions:-

a) Show that the set of all real convergent sequences is a vector space over the field of real number.

b) For what values of 'C', the real values function:- $f(x) = \frac{c}{1+(x-\theta)^2}, -\infty < x < \infty$, a probability density function for random variable X?

c) If nullity of the matrix $A = \begin{bmatrix} k & 1 & 2 \\ 1 & -1 & -2 \\ 1 & 1 & 4 \end{bmatrix}$ is 1, then find k?

d) Let A be an 8×8 matrix with eigenvalues 1, -1, 0. Then, find the determinant of $I+A^{100}$.

e) Define a random variable and its mathematical expectation.

f) Give an example of infinite dimensional vector space.

g) Define Poisson distribution and state conditions under which this used.

h) Show that Poisson distribution is a limiting case of binomial distribution.

i) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

j) Let P and Q be two matrices of order 4× 6and 5×4. If rank (Q) = 4 and rank (QP) = 2, then find the rank of P. (10×1)

UNIT – I

II. a) Find the values of λ and μ for which the system of equations:-
$x + 2y + z = 6,\ x + 4y + 3z = 10,\ x + 4y + \lambda z = \mu$, has a:-
i) Unique solution
ii) Infinite number of solutions
iii) No solution

b) For what values of 'k' do the following set of vectors form a basis in R^3:-
$\{(k, 1-k, k), (0, 3k-1, 2), (-k, 1, 0)\}$. (5,5)

III. a) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ x & y & z \end{bmatrix}$ and let V = $\{(x, y, z) \epsilon R^3$ such that det (A) = 0}, then, find the dimension of the vector space.

b) For what values of 'a' and 'b', the matrix:- $A = \begin{bmatrix} 0 & a & 0 \\ b & 0 & b \\ 0 & a & 0 \end{bmatrix}$ is diagonalizable? (5,5)

IV. a) State rank-nullity theorem. Consider the matrix mapping $A: R^4 \to R^2$, where:-

$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$. Find a basis and dimension of :-

i) Image of A
ii) Kernel of A
Also verify rank-nullity theorem.

b) For each of the following linear operations L on R^2, find the matrix A that represents L relative to the usual basis of R^2:-
i) L is defined by L(1,0) = (2,4) and L (0,1) = (5,8)
ii) L is the rotation in R^2 counter clock wise by 90°.
iii) L is the reflection in R^2 about the lim e y = -x (5,5)

P.T.O.

UNIT – II

V. a) In a bolt factory, machines A, B, C manufactures 25%, 35% and 40% of the total output. Of their output 5, 4, 2 percent are known to be defective bolts. A bolt is drawn at random from the product and is found to be defective. What are the probabilities that was manufactured by:-

i) Machine A

ii) Machine B or C

b) Suppose X has the density function

$f(x) = \{3x^2, \quad 0 < x < 1.$

$\quad 0, \quad \text{otherwise}$

What is the expected area of a random isosceles right angle triangle with hypotenuse X? (5,5)

VI. a) Let the probability density function of a random variable X be:-

$f(x) = \{630x^4\,(1-x)^4, \quad 0 < x < 1$

$\quad 0, \quad \text{otherwise}$

What is the exact value of $P\,(|X-\mu| < 2\sigma)$?

What is the approximate value of $P\,(|X-\mu| \le 2\sigma)$ when one uses Chebyshev inequality?

b) Suppose that 2000 points are selected independently and at random from the unit sphere $S = \{(x, y)|0 \le x, y \le 1\}$. Let X equal the number of points that fall in $A = \{(x, y)|x^2 + y^2 < 1\}$. How is X distributed? What are the mean, variance and standard deviation of X? (5,5)

VII. a) If $X \sim N(25,36)$,where N denotes the normal distribution, then what is the value of the constant 'C' such that $P = (|X-25| \le C) = 0.9544$?

b) State central-limit theorem. A population consists of five numbers 2, 3, 6, 8, 11. Consider all possible samples of size two which can be drawn with replacement from this population. Calculate the standard error of sample mean. (5,5)

0-0-0