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1019

B.E. (Electronics and Communication Engineering)
Fourth Semester
MATHS-405: Algebra and Complex Analysis

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.

x-x-x

Question I (a) Express $v = (3, 7, -4)$ in $\mathbb{R}^3$ as a linear combination of the vectors $u_1 = (1, 2, 3)$, $u_2 = (2, 3, 7)$, $u_3 = (3, 5, 6)$.

(b) Find the row rank of the matrix $A = \begin{bmatrix} 1 & -3 & 6 & 0 \\ 4 & 11 & 0 & -3 \\ 5 & 1 & 9 & -3 \\ 2 & 4 & 9 & 0 \end{bmatrix}$

(c) Define the rank and nullity of linear mapping. What is the relation between them.
(d) Define pole and residue at pole of a function of complex variable.
(e) Find all the n $n-$th roots of unity.

$(2 \times 5 = 10)$

Part A

Question II (a) Find the inverse of the following matrix using Gauss-Jordan method:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

(b) Solve the following system of linear equations.

$$7x_1 + 2x_2 - 2x_3 - 4x_4 + 3x_5 = 8$$
$$-3x_1 - 3x_2 + 2x_4 + x_5 = -1$$
$$4x_1 - x_2 - 8x_3 + 20x_5 = 1$$

(5+5=10)

Question III (a) State the Cayley-Hamilton theorem. Using it, invert the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 0 & -1 \end{bmatrix}.$$

(b) When is a set of vectors said to be linearly independent? Prove that if any set of vectors contains the zero vector, then that set is necessarily linearly dependent. What can be said about the following set of vectors:

$$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$$

(5+5=10)

Question IV (a) Let $F : \mathbb{R}^4 \to \mathbb{R}^3$ be the linear mapping defined by $F(x, y, z, t) = (x - y + z + t, x + 2z - t, x + y + 3z - 3t)$. Find a basis and the dimension of the image of F and the kernel of F.

(b) Find the eigen values and eigen vectors of the following matrix A. Also find the geometric and algebraic multiplicity of the eigen values. Is the matrix diagonalizable? Justify.

$$A = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 2 & 0 & -2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}.$$

(5+5=10)

P.T.O.

Part B

Question V (a) Show that the function $u(x, y) = e^{-2xy}\cos(x^2 - y^2)$ is harmonic. Hence find a function $v(x, y)$ such that the complex valued function $f(z) = u(x, y) + \iota v(x, y)$ is entire.

(b) When is a complex valued function $f(z)$ of a complex variable z said to be analytic? Find the regions in the complex plane where the following functions are analytic:
(i) $f(z) = | z |$ (ii) $f(z) = \text{Re } z/\text{Im } z$.

(5+5=10)

Question VI (a) Let u and v denote the real and imaginary components of the function f defined by the equations

$$f(z) = \begin{cases} \dfrac{(\overline{z})^2}{z} & \text{when } z \neq 0, \\ 0 & \text{when } z = 0. \end{cases}$$

Verify that the Cauchy-Riemann equations are satisfied at the origin $z = 0$ but $f'(0)$ nevertheless fails to exist.

(b) Solve the following equations in complex variable z
(i) $e^z = 1 + i$
(ii) $\cosh z = \dfrac{1}{2}$

(5+5=10)

Question VII (a) Expand $f(z) = \dfrac{1}{(z+1)(z+3)}$ in Laurent's series valid for the region:

(*i*) $1 < |z| < 3$, (*ii*) $0 < |z + 1| < 2$.

(b) What the image of first quadrant in the $z-$plane by the mapping $f(z) = z^2$ in the $w-$plane? Explain.

(5+5=10)

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