Exam.Code:0922
Sub. Code: 6882

1019

B. E. (Information Technology)

Fourth Semester

ITE-402: Discrete Mathematics (Old)

(Batch 2016-17)

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.

x-x-x

Question I (a) A committee of eight is to be formed from 16 men and 10 women. In how many ways can the committee be formed so that there are even number of women.

(b) Let R be a non-empty relatioh on a collection of sets defined by ARB if and only if $A \cap B = \phi$. Then which of the following is correct and why?

(i) R is reflexive and transitive

(ii) R is an equivalence relation

(iii) R is symmetric and not transitive

(iv) R is not reflexive and not symmetric

(c) What is the correct translation of the following statement into mathematical logic? "Some real numbers are rational"

(d) Give a closed formula for the generating function of the sequence 1,3,9,27,...

(e) Define Eulerian cycle and Eulerian trail for a graph.

(2 × 5 = 10)

Part A

Question II (a) Establish the validity of the following argument by using method of proof:

$p \to (q \wedge r)$

$r \to s$

$\sim (q \wedge s)$

$\therefore \sim p$

(b) Which of the following functions are one-one? Explain. Also determine the range of these functions.

(i) $f : \mathbb{Q} \to \mathbb{Q},\ f(x) = 2x$

(ii) $f : [-\pi/2, \pi/2] \to \mathbb{R}, f(x) = \cos x$

(5+5=10)

Question III (a) For $A = \mathbb{R} \times \mathbb{R}$, define relation R on A by $(x_1, y_1)R(x_2, y_2)$ if $x_1 = x_2$. Show that R is an equivalence relation on A. Describe geometrically the equivalence classes and partitions of A induced by R.

(b) Prove the following version of the Pigeon Hole Principle: Suppose there are n pigeon holes. To ensure that at least one pigeon hole contains at least k pigeons, the total number of pigeons must be at least $m = n(k-1) + 1$.

(5+5=10)

P.T.O.

Question IV (a) What is a partially ordered set? What is a totally ordered (or linearly ordered) set? Give an example for both.

(b) Let R be the following relation on $A = \{1, 2, 3, 4, 5\}$:

$$R = \{(1,2), (2,4), (3,2), (3,5), (4,4), (5,2), (5,4)\}.$$

(a) Find the matrix M_R of R. (b) Find the domain and range of R. (c) Find R^{-1}.
(d) Draw the directed graph of R. (e) Find the composition relation $R \circ R$

(5+5=10)

Part B

Question V (a) Find the number of integers between 1 and 10000, both inclusive, which are divisible by none of 5,6 or 8.

(b) Find the number of ways of placing 20 similar balls into 6 numbered boxes so that the first box contains any number of balls between 1 and 5 inclusive and the other 5 boxes must contain 2 or more balls each. Use generating functions.

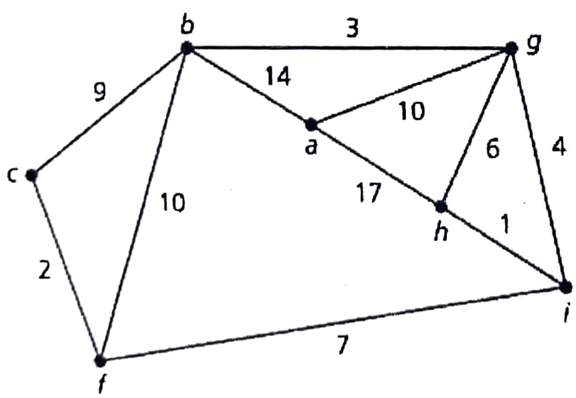
(5+5=10)

Question VI (a) Find the generating function for the number of partitions of a positive integer n into **distinct** summands.

(b) Solve the recurrence relation $a_n - 3a_{n-} = 5(3^n)$, where $n \geq 1$ and $a_0 = 2$.

(5+5=10)

Question VII (a) Apply Dijkstra's algorithm to the weighted graph below to find shortest distance from vertex c to each of the other six vertices in the graph.



(b) Explain with help of examples, the following terms in respect of a graph:
(i) Degree of a vertex
(ii) Discrete graph
(iii) Complete graph
(iv) Subgraph of a graph
(v) Connected graph

(5+5=10)

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