Exam.Code:0906
Sub. Code: 6660

1019

B.E. (Bio-Technology)
Second Semester
MATHS-201: Differential Equations and Transforms
(Common to all streams)

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part. Use of non-programmable calculator is allowed.

x-x-x

1. (a) Find the Laplace transform of $\frac{\sin t}{t}$. (5 × 2 = 10)

   (b) Define even and odd functions and write the corresponding Fourier series for these functions with period $p = 2L$.

   (c) Formulate the partial differential equation by eliminating the arbitrary constants: $z = a(x+y) + b(x-y) + abt + c$ where z is a function of three independent variables x, y, t and a, b, c are arbitrary constants.

   (d) Solve the differential equation: $(x-y)^2 \frac{dy}{dx} = 1$.

   (e) Define unit step function and find its Laplace transform.

PART A

2. (a) Solve the following differential equations: (3+3)

   (a) $y' + y = \frac{1}{1+e^{2x}}$.

   (b) $y'''' - 4y''' + 14y'' - 20y' + 25y = 0$

   (b) Find the inverse Laplace transform of $\ln \frac{s+1}{s-1}$. (4)

3. (a) Find the general solution of the ordinary differential equation (5)

$$(D^3 - 2D + 4)y = x^2 + e^{3x}$$

   (b) Find the general solution of the differential equation using method of variation of parameters: (5)

$$(D^2 + 9)y = \sec 3x$$

4. (a) Find the inverse Laplace transform of (5)

$$\frac{se^{-s/2} + \pi e^{-s}}{s^2 + \pi^2}$$

   (b) Evaluate $L\left[e^{-4t}\int_0^t \frac{\sin 3u}{u}\,du\right]$. (5)

P.T.O.

PART B

5. (a) Find the Fourier cosine transform of $f(x) = e^{-ax}, a > 0$. (5)

   (b) Find the Fourier series of the periodic function $f(x) = x^2$, if $-1 < x < 1$ with $p = 2$. (5)

6. (a) Find the general solution of the partial differential equation (5)

$$2x(y + z^2)\,p + y(2y + z^2)\,q = z^3.$$

   (b) Let $f(x) = \pi + |x|$ $(-\pi < x < \pi)$ be a periodic function with period 2π. Find the Fourier series for $f(x)$. (5)

7. (a) Find the temperature $u(x,t)$ in a bar of length L. Both the ends of the bar are kept at temperature zero and the initial temperature of the bar is given by $u(x,0) = f(x)$. The one-dimensional heat equation is given by (10)

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad c^2 = \frac{K}{\sigma \rho}$$

x-x-x