1129

B. Engg. (Biotechnology) 3rd Semester

MATHS-302: Linear Algebra and Operations Research (Common with FT)

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wed: 3 Hours

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Max. Marks: 50

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 $(2x2\frac{1}{2})$

Attempt <u>five</u> questions in all, including Q. No. I which is compulsory and selecting two questions from each Unit. Use of simple calculator is allowed.

- (a) Define the rank of the matrix. Find the same for the matrix $A = \begin{bmatrix} 8 & 0 & 0 & 16 \\ 0 & 0 & 0 & 6 \\ 0 & 9 & 9 & 9 \end{bmatrix}.$
 - (b) Define a vector space and basis of a vector space with suitable examples.
 - (c) Prove that similar matrices have the same eigenvalues.
 - (d) Define (i) convex set, (ii) hyper planes with suitable examples.
 - (e) State the objective of network analysis. Define total float and free float. (5×2)

UNIT-I

- II. (a) Determine whether or not vectors (1,-2.1), (2,1,-1), (7,-4,1) in R³ are linearly independent or not? If linearly dependent, then find the relation between them.
 - (b) Show that the set of all 2×2 matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition denoted by $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication $k \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$ is a vector space.
 - (c) Show that the set $s = {\vec{v}_1, \vec{v}_2, \vec{v}_3}$ is a basis of R^3 , where $\vec{v}_1 = (1,2,1), \ \vec{v}_2 = (2,9,0), \ \vec{v}_3 = (3,3,4)$ (3+4+3)
- III. (a) Find the inverse of the matrix using Gauss-Jordan method: $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 3 & 2 \\ 2 & 4 & 3 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix}$
 - (b) Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ and hence find A^4 . (5+5)
- IV. (a) Use graphical method to solve: Maximize $z = 3x_1 + 2x_2$ Subject to the constraints: $-2x_1 + 3x_2 \le 9$, $x_1 - 5x_2 \ge -20$, $x_1, x_2 \ge 0$
 - (b) Use Charne's M method to solve: Maximize $z = x_1 + 2x_2 + 3x_3 - x_4$ Subject to the constraints: $x_1 + 2x_2 + 3x_3 = 15$, $2x_1 + x_2 + 5x_3 = 20$, $x_1 + 2x_2 + x_3 + x_4 = 10$, $x_1, x_2, x_3, x_4 \ge 0$ (5+5)

P.T.O.

UNIT-II

V. (a) Consider the problem:

Maximize $z = 2x_2 - 5x_3$

Subject to the constraints: $x_1 + x_3 \ge 2$, $2x_1 + x_2 + 6x_3 \le 6$,

 $x_1 - x_2 + 3x_3 = 0$, $x_1, x_2, x_3 \ge 0$

(i) Write its dual.

(ii) Solve the primal and then find the solution of the dual

(b) Use dual simplex method to solve:

Maximize $z = -2x_1 + x_2$

Subject to the constraints: $3x_1 + x_2 \ge 3$, $4x_1 + 3x_2 \ge 6$, $x_1 + 2x_2 \ge 3$, $x_1, x_2 \ge 0$

VI. (a) Determine an initial basic feasible solution to the following transportation problem using:

(i) North-West corner cell method,

(ii) Vogel's approximation method

Destination

Supply $\begin{array}{c|c}
E_1 \\
\hline
7 \\
1 \\
12
\end{array}$ 8
9

 A_1 B_1 C_1 Di A 11 10 3 Origin В 1 4 7 2 C 9 4 8 12 3

(b) Solve the following assignment problem:

| 1 | | | 1 | |
|----------------|-------|-------|-------|-------|
| | M_1 | M_2 | M_3 | M_4 |
| J_1 | 5 | 8 | 3 | 2 |
| J ₂ | 10 | 7 | 5 | 8 |
| J_3 | 4 | 10 | 12 | 10 |
| J_4 | 8 | 6 | 9 | 4 |
| | | | | |

(5+5)

VII. (a) Distinguish between PERT and CPM. What is a critical path?

(b) A small project is composed of seven activities whose time estimates are listed in the table as follows:

| Activity | | Estimated Duration (weeks) | | | |
|----------|---|----------------------------|-------------|-------------|--|
| 1 | j | Optimistic | Most Likely | Pessimistic | |
| 1 | 2 | 1 | 1 | 7 | |
| 1 | 3 | 1 | 4 . | 7 | |
| 1 | 4 | . 2 | 2 | 8 | |
| 2 | 5 | 1 | 1 | 1 | |
| 3 | 5 | 2 | 5 | 14 | |
| 4 | 6 | 2 | 5 | 8 | |
| 5 | 6 | 3 | 6 | 15 | |

(i) Draw the project network.

(ii) Find the expected duration and variance of each activity. What is expected project length?

(iii) Calculate the variance and standard deviation of project length. What is the probability that the project will be completed at least 4 weeks earlier than expected? (5+5)

3 Hours

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Answer the f

a) Total

b) Facto

c) Expla

d) Limit

e) Facto

f) Natio

g) Marg

h) Inferi

i) Total

i) Total

What is the explain the v

a) Give relati

b) What is m

N. Explain indif schedule and

V. Define nation

VI Explain ma competition.

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