Exam. Code: ~~0933~~ 0933
Sub. Code: 6974

1129

B. Engg. (Electrical and Electronics Engg.)

3rd Semester

MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt __five__ questions in all, including Q. No. I which is compulsory and selecting atleast two questions each from Unit I-II. Use of simple calculator is allowed.*

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I. (a) Define subspace of a vector space. Prove that $w = \{(x, y) \mid x = 3y\}$ is a subspace of R^2.

(b) Find basic for column space of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 0 \end{bmatrix}$.

(c) Check $T : R^2 \rightarrow R^3$, where $T(x, y) = (x + y, x - y, y)$ is linear transformation or not.

(d) Define Kernel and image of a linear mapping with a suitable example.

(e) Prove that similar matrices have the same eigenvalues.

(f) Prove that $f(z) = \bar{z}$ is not analytic at any point.

(g) Find the image of the circle $|z| = \lambda$ under the transformation w=5z.

(h) Locate and classify the singular points of $f(z) = \frac{z - \sin z}{z^3}$.

(i) Find the pole and residue at the pole of the function $f(z) = \frac{z}{z-1}$.

(j) For what values of constant k does the system: x-y=3, 2x-2y=k, have no solution, exactly one solution and many solutions. (10×1)

UNIT-I

II. (a) Prove that the vectors (2,3,0), (2,1,0) and (8,13,0) are the early dependent in R^3. Find the relation between them.

(b) Find the rank of the matrix: $A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$.

(c) Find the values of μ for which the system of equations: $x + y + z = 1$, $x + 2y + 3z = \mu$, and $x + 5y + 9z = \mu^2$ will be consistent. For each value of μ obtained, find the solution f the system. (3+3+4)

III. (a) Does the vector $\bar{b} = (3,4,5)'$ lie in the column space of $A = \begin{bmatrix} 1 & 2 & -1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$? Justify.

(b) Prove that the set of vectors $\{(2,1,4),(1,-1,2),(3,1,-2)\}$ forms a basis of R^3.

(c) Prove that linear transformation $T : R^2 \rightarrow R^2$ defined by $T(x_1, x_2) = (x_1 \cos\theta + x_2 \sin\theta, -x_1 \sin\theta + x_2 \cos\theta)$ is a vector space isomorphism's. (3+3+4)

P.T.O.

IV. (a) Consider the matrix mapping $A: R^4 \to R^3$, where $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$. Find a basis and the dimension of (i) the image of A, (ii) the Kernel of A. Also verify Rank-Nullity theorem.

(b) Find the matrix P that diagonalizes $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$. Also determine $P^{-1}AP$.

(c) Consider the linear operator T on R^2 defined by $T(x, y) = (5x + y, 3x - 2y)$ and the following bases of R^2: $s = \{(1,2),(2,3)\}$ *and* $s^{-1} = \{(1,3),(1,4)\}$.

(i) Find the matrix A representing T relative to the basis S.

(ii) Find the change of basis matrix P from S to S^1. (4+3+3)

UNIT-II

V. (a) Find the values of z such that $z = e^{\frac{\pi}{3}}$.

(b) Find the constants a,b,c such that the function: $f(z) = -x^2 + xy + y^2 + i(ax^2 + bxy + cy^2)$ is analytic. Express f(z) in terms of z.

(c) Prove that the function $u(r,\theta) = r^2$ is harmonic. Find its conjugate harmonic and the corresponding function f(z). (3+3+4)

VI. (a) Find the Laurent's expansion for $f(z) = \dfrac{7z-2}{z^3 - z^2 - 2z}$ in the following regions:

(i) $0 < |z+1| < 3$ (ii) $1 < |z+1| < 3$

(b) Evaluate $\int_C \dfrac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$, when C is the curve $|z| = 3$. (5+5)

VII. (a) Evaluate $\int_0^{2\pi} \dfrac{\cos 3\theta}{5 - 4\cos\theta} d\theta$ using complex integration.

(b) Find two bilinear transformations whose fixed points are 1 and 2. (5+5)

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