2|12|19 (M)

Exam.Code:0921
Sub. Code: 6831

1129

B. E. (Information Technology)

Third Semester

MATH-303: Linear Algebra and Probability Theory

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt <u>five</u> questions in all, including Question No. I which is compulsory and selecting two questions from each Section.*

x-x-x

1 (a) Is the map $F : \mathbf{R}^2 \longrightarrow \mathbf{R}^3$ defined by $F(x, y) = (x + 3, 2y, x + y)$ linear ? Justify. (2)

(b) Find a 2×2 singular matrix which maps $(2, -4)$ and $(-1, 2)$ into $(1, 1)$ and $(1, 3)$.(2)

(c) What is the probability of the occurrence of a number that is odd or less than 5, when a fair die is rolled. (2)

(d) A loss for a company has moment-generating function $M(t) = 0.16/(0.16 - t)$, $t < 0.16$. An insurance policy pays a benefit equal to 70% of the loss. What is the moment-generating function of the benefit? (2)

(e) Suppose v is an eigen vector of A corresponding to eigen value λ. Prove that $f(\lambda)$ is an eigen value of $f(A)$, for any polynomial $f(t)$. (2)

SECTION-A

2 (a) Find the basis and dimension of $U+W$, where U = span $\{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$ and W = span $\{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$. (5)

(b) Find a linear map $F : \mathbf{R}^3 \longrightarrow \mathbf{R}^3$ whose image is spanned by $(1, 2, 3) and (4, 5, 6)$. (5)

3 (a) Solve the system of equations $x + 2y - 3z = 2, 2x - 3y + 8z = 7, 3x - 4y + 13z = 8$. (5)

(b) Express $(2, -5, 3) \in \mathbb{R}^3$ as a linear combination of vectors $(1, -3, 2), (2, -4, -1)$ and $(1, -5, 7)$. (5)

4 (a) Find the matrix B that represents the linear map $F : \mathbf{R}^4 \longrightarrow \mathbf{R}$ defined by $F(x, y, z, t) = 2x + y - 7z - t$ relative to the standard bases for $\mathbf{R}^4$ and $\mathbf{R}$. (5)

(b) Check whether the matrix $A = \begin{pmatrix} 2 & 1 & -2 \\ 2 & -3 & -4 \\ 1 & 1 & -1 \end{pmatrix}$ is diagonalizable or not. Also find corresponding transforming matrix. (5)

SECTION-B

5 (a) Three urns of the same appearance have the following proportion of balls.

First urn: 2 black	1 white
Second urn: 1 black	2 white
Third urn: 2 black	2 white.

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One urn is selected and one ball is drawn. It turns out to be white. What is the probability of drawing a white ball again, the first one not having been returned. (5)

(b) A biased coin with $P(H) = 0.4$ is tossed 100 times. Let X be the number of heads in the 100 tosses. Use Chebyshevs inequality to find an upper bound for $P(X = 30 \text{ or } X = 50)$ (5)

6 (a) A random variable X has the density function

$$f(X) = \frac{c}{X^2 + 1}, \quad -\infty < X < \infty.$$

Find the probability that X^2 lies between 1/3 and 1. (5)

(b) A little deck has 6 cards: 2 Aces, 2 Kings, and 2 Queens. Two cards are drawn at random, without replacement. If X is the number of Aces obtained, find E(X). (5)

7 (a) If X and Y are independent Poisson variables such that $P(X= 1)= P(X= 2)$ and $P(Y= 2)= P(Y= 3)$. Find the variance of X - 2Y. (5)

(b) The marks obtained by a number of students for a certain subject are assumed to be approximately normally with mean value 65 and standard deviation of 5. If three students are taken at random from this set, what is the probability that exactly two of them will have marks over 70? (5)

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