: 6946

Exam. Code: 0907
Sub. Code: 6948

1129

B.E. (Biotechnology) Third Semester

AS-306: Engineering Maths – III

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.

x-x-x

1 (a) Does alternating harmonic series converge? State Leibnitz test. State absolute and conditional convergence of alternating series with suitable examples.

(b) What is the difference between Taylor and Maclaurin series ? Does Taylor series always converge to its generating function? $(5\times 2=10)$

(c) Define linearly independent, dependent vectors and rank of a matrix? Find the rank for the matrix: $A=\begin{bmatrix}1 & 4\\ 2 & 8\end{bmatrix}$.

(d). Explain the difference between differentiability and analyticity. Find the regions in the complex plane where the following functions are analytic: (i) $f(z)=|z|^2$, (ii) $f(z)=\frac{\text{Re}(z)}{\text{Im}(z)}$?

(e). Define bilinear and isogonal transformations. Give one example of each.

PART-A

2. (a) Examine the convergence or divergence of the following sequences:

(i) $a_n=\frac{1}{\sqrt{n^2-1}-\sqrt{n^2+n}}$, (ii) $a_n=\frac{(\ln n)^5}{\sqrt{n}}$, (iii) $a_n=\sqrt[n]{n^2+n}$, (iv) $a_n=\sinh n$.

(b) Examine the convergence or divergence of the following series:

(i) $\sum_{n=1}^{\infty}\frac{3^{n-1}-1}{6^{n-1}}$, (ii) $\sum_{n=1}^{\infty}\frac{e^n}{1+e^{2n}}$, (iii) $\sum_{n=1}^{\infty}\frac{3^{n-1}-1}{6^{n-1}}$ (iv) $\sum_{n=1}^{\infty}\frac{\ln n}{n^3}$.

3. (a) Find the radius and interval of convergence for the series: $\sum_{n=0}^{\infty}\frac{(x-\sqrt{2})^{2n+1}}{2^n}$.

For what values of x does the series converge (i) absolutely, (ii) conditionally?

P.T.O.

(b) State Cayley-Hamilton theorem. Verify it for the matrix: $A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$.

Also compute A^{-1}.

4. (a) Find the eigenvalues and the corresponding eigenvectors of the matrix: $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & 1 \\ 2 & -1 & 3 \end{bmatrix}$.

(b) Diagonalize the matrix: $A = \begin{bmatrix} 2 & 1-2i \\ 1+2i & -2 \end{bmatrix}$. $(4+3+3)$

(c) Examine whether or not the given matrices are similar: $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$?

PART-B

5. (a) Prove that the function $f(z)$ defined by $f(z) = \sqrt{|xy|}$ satisfies Cauchy-Riemann equations at the origin but it is not analytic as this point.

(b) Prove that if $f(z) = u + iv$ is analytic in a domain D, then the real valued functions $u(x, y)$ and $v(x, y)$ satisfies the Laplace equation.

6. (a) State Laurent's series. Expand $f(z) = \dfrac{1}{(z-1)(z-2)}$ in Laurent's series valid for the region:

(i) $1 < |z| < 2$. (ii) $0 < |z-1| < 1$.

(b) State Cauchy residue theorem. Use it to evaluate the integral $I = \int_C \dfrac{e^z - 1}{z(z-1)(z-i)^2} dz$, where C is the circle $|z| = 2$.

7. (a) Discuss the mapping $w = \sinh z$.

(b) Evaluate the integral $I = \int_0^{2\pi} \dfrac{1}{2 - \sin\theta} d\theta$ by complex integration.

x-x-x