Exam. Code: 0939
Sub. Code: 7045

1129

B. Engg. (Mechanical Engg.)

3rd Semester

AS-301: Engineering MATH-3

Time allowed: 3 Hours | Max. Marks: 50

NOTE: *Attempt **five** questions in all, including Q. No. I which is compulsory and selecting atleast two questions from each Unit*

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I. (a) Define limit of a sequence? When a sequence is said to be convergent and bounded? Explains the relation between sequence and series together with their practical applications.

(b) State Taylor's series and Taylor's formula with remainder term. Explain the importance of Taylor's expansion.

(c) Explain the difference between Gauss elimination and Gauss-Jordan method. Why do we apply pivoting in these methods and which one superior in solving linear system of equation?

(d) Prove that $w = \cos z$ is not a bounded function.

(e) Define conformal mapping with a suitable example. How it is different from isogonal mapping? (5×2)

UNIT-I

II. (a) Discuss the convergence or divergence of the following series: -

(i) $\sum_{n=1}^{\infty}\left(\frac{1}{\sqrt{n}}-\frac{1}{\sqrt{n+1}}\right)$ (ii) $\sum_{n=1}^{\infty}\frac{8\tan^{-1}n}{1+n^2}$ (iii) $\sum_{n=1}^{\infty}\frac{\ln n}{n\frac{3}{2}}$

(b) Define power series and its radius of convergence. Find radius and interval of convergence for the series $\sum_{n=1}^{\infty}\frac{(4x-5)^{2n+1}}{n^{3/2}}$. For what values of x does the series converge (i) absolutely (ii) conditionally? (5+5)

III. (a) Find the Maclaurin series for six. Discuss its convergence. For what values of x can we replace $\sin x$ *by* $x-\frac{x^3}{\underline{3}}$ with an error of magnitude no greater than 3×10^{-4}?

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(b) Define the rank of the matrix. Hence, find the same for $A=\begin{bmatrix}1 & 3 & 2 & 2\\ 1 & 2 & 1 & 3\\ 2 & 4 & 3 & 4\\ 3 & 7 & 4 & 8\end{bmatrix}$. (5+5)

IV. (a) Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix}3 & 0 & 0\\ 5 & 4 & 0\\ 3 & 6 & 1\end{bmatrix}$.

(b) State Cayley-Hamilton theorem. Verify it for the matrix $A=\begin{bmatrix}1 & 2 & 0\\ -1 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$. Also compute A^{-1} using it. (5+5)

UNIT-II

V. (a) Find all values of z such that $e^{\frac{1}{z}}=1-i$.

(b) Test the continuity at z=0 if $f(z)=\begin{cases}\dfrac{\mathrm{Im}.z}{|z|}, & z\neq 0\\ 0, & z=0\end{cases}$.

(c) Find the principal value of i^i. (3+4+3)

VI. (a) Define an analytic function and prove that an analytic function of constant modulus is constant.

(b) State Laurent's expansion. Find the Laurent's series expansion for $f(z)=\dfrac{1}{z-z^3}$ *in* $1<|z+1|<2$. (5+5)

VII. (a) Find the sum of the residues of the function $f(z)=\dfrac{\sin z}{z\cos z}$ at its pole inside $|z|=2$.

(b) Evaluate $\int_0^{2\pi}\dfrac{d\theta}{5+4\cos\theta}$ using residue integration theorem. (5+5)

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