Exam.Code:1014
Sub. Code: 7749

1129

M.E. (Mechanical Engineering)
First Semester
MME-101: Advanced Engineering Mathematics

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt <u>five</u> questions in all, selecting atleast two questions from each Part.*

x-x-x

PART A

1. (a) Solve the following differential equation by power series method: (5)

$$(x^2 + 1)y'' + xy' - xy = 0$$

(b) Using recurrence relatios, show that (5)

$$4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x)$$

2. (a) Prove that $\int_{-1}^{1} P_m(x) \cdot P_n(x)dx = 0,\ n \neq m.$ (5)

(b) Find a basis of solutions by Frobenius method: (5)

$$xy'' + 5y' + xy = 0$$

3. (a) Find a general solution in terms of J_ν and $J_{-\nu}$ or indicate if not feasible: (5)

$$x^2y'' + xy' + (x^2 - \frac{1}{16})y = 0$$

(b) Prove that $J_n(x)$ is the coefficient of z^n in the expression of $e^{\frac{x}{2}\left(z - \frac{1}{z}\right)}$. (5)

4. (a) Find the eigen values and eigen functions of the following differential equation: (5)

$$y'' - 2y' + (\lambda + 1)y = 0,\quad y(0) = 0,\ y(1) = 0$$

(b) Show that $P_{2n}(0) = (-1)^n \frac{1.3.5...(2n-1)}{2.4.6...2n}$. (5)

PART B

5. Use the Runge-Kutta fourth order method to find $y(0.2)$ with $h = 0.1$ for the initial value problem (10)

$$\frac{dy}{dx} = \sqrt{x - y},\quad y(0) = 1$$

6. Using Picard's method, solve the differential equation: (10)

$$\frac{dy}{dx} = x - y,$$

given that $y(0) = 1$ and find $y(0.2)$ to five decimal places.

7. Find the values of $u(x,t)$ satisfying the parabolic equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(0,t) = 0 = u(5,t)$ and $u(x,0) = 5x - x^2$ at the points: (10)

$$x = i:\ i = 0, 1, 2, 3, 4, 5 \quad \text{and} \quad y = j:\ j = 0, 1, 2, 3, 4, 5.$$

8. Solve $16\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 y}{\partial t^2}$ taking $h = 1$, upto $t = 1.25$, under the conditions $u(0,t) = u(5,t) = 0,\ u_t(x,0) = 0$ and $u(x,0) = x^2(5 - x)$. (10)

x-x-x