Exam.Code:0905
Sub. Code: 6641

1129

B.E. (Bio-Technology) First Semester
MATHS-101: Calculus
(Common to all streams)

Time allowed: 3 Hours **Max. Marks: 50**

NOTE: *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Unit.*

x-x-x

I. a) Find the constant a so that the vector $\vec{V} = (x + 3y)\,\hat{i} + (y - 2z)\,\hat{j} + (x + az)\,\hat{k}$ is solenoidal.

b) What does the ratio and root test say about Convergence of Infinite positive series?

c) Find v_x in terms of u and v if the equations $x = v\ ln\ u$ and $y = u$ ln v define u and v as functions of independent variables x and y.

d) State Gauss Divergence theorem?

e) Discuss the convergence of : $\sum_{n=1}^{\infty} \frac{5^n}{3+4^n}$. (5x2)

UNIT – I

II. a) Obtain the quadratic approximation to the function $f(x,y) = xy^2 + y\cos(x - y)$ about the point(l, 1). Find the maximum absolute error in the region) $|x - 1| < 0.05$, $|y - 1| < 0.1$.

b) Find the circumference (Length) of circle $x^2 + y^2 = r^2$. (5,5)

III. a) Discuss the convergence of the following:-

i) $\{a_n\}$ where $a_1 = \sqrt{5}$ and $a_{n+1} = \sqrt{5 + a_n}$

ii) $\sum a_n$ where $a_n = \begin{cases} (n/2^n) & \text{if } n \text{ is a odd number} \\ (1/2^2), & \text{if } n \text{ is a Even number} \end{cases}$

b) Find the interval and radius of convergence for $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x+2)^n}{n2^n}$ and also find the values of x for which series converges conditionally. (5,5)

IV. a) Examine the functional dependence of the functions u = $\frac{x+y}{x-y}$ *and* = $\frac{xy}{(x-y)^2}$. If they are dependent, find the relation between them.

b) Find the volume of solid generated by revolving the region bounded by the curves $y = -1$ and $y = 3 - x^2$ about the line y = -1. (5,5)

P.T.O.

UNIT – II

V. a) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}$ dxdy and hence evaluate $\int_0^\infty e^{-x^2} dx$

b) Define Binomial vector. Find Binomial vector for $r(t) = t\hat{i} + (a \cos(t/a))\,\hat{j}$, $a > 0$.

(5,5)

VI. a) Find curvature and torsion for the curve $r(t) = (a\cos \alpha t)\hat{i} + (a\sin \alpha t)\hat{j} + bt\hat{k}, \alpha, a, b > 0,$ $\alpha^2 a^2 + b^2 = 1.$

b) The derivative of $f(x, y)$ at $P_o(x_o, y_o)$ in the direction of $(3\hat{i} + 4\hat{j})/5$ is 1 and in the direction of $(4\hat{i} - 3\hat{j})/5$ is 3. Find a unit vector $\hat{b}$ such that (i) $D_b f(P_o)$ is maximum and (ii) $D_b f(P_o)$ is minimum.

(5,5)

VII. a) Using Green's theorem, find the counterclockwise circulation of $\vec{F} = (y + e^x \ln y)\hat{i} + (e^x / y)\vec{j}$ around the boundary of the region that is bounded above by the curve $y = 3 - x^2$ and below by the curve $y = x^4 + 1$.

b) State Stoke's theorem and verify it for the vector field $\vec{F} = (3x - y)\hat{i} + -2yz^2\hat{j} - 2y^2 z\hat{k}$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 16, z > 0$.

(5,5)

x-x-x