

1108
M.E. Electrical Engineering (Power Systems)
1st Semester
EE-8103: Optimization Techniques

Time allowed: 3 Hours

Max. Marks: 50

Note: Attempt five questions in all.

x-x-x

1. (a) Consider the following linear programming problem and its optimal table below: (5)

$$\min z = -2x_1 - 4x_2 - 2.5x_3$$

subject to $3x_1 + 4x_2 + 2x_3 \leq 60$, $2x_1 + x_2 + 2x_3 \leq 40$, $x_1 + 3x_2 + 3x_3 \leq 30$, $x_1, x_2, x_3 \geq 0$
Here, x_4, x_5, x_6 are slack variables.

| Basic variables | x_1 | x_2 | x_3 | x_4 | x_5 | x_6 | Constant |
|-----------------|-------|-------|-------|-------|-------|-------|----------|
| x_1 | 1 | 0 | -6/5 | 3/5 | 0 | -4/5 | 12 |
| x_5 | 0 | 0 | 3 | -1 | 1 | 1 | 10 |
| x_2 | 0 | 1 | 7/5 | -1/5 | 0 | 9/15 | 6 |
| $-z$ | 0 | 0 | 7/10 | 2/5 | 0 | 4/5 | 48 |

Suppose the coefficient of x_2 in the second constraint is changed from 1 to 2. Use sensitivity analysis to find the new optimal solution.

- (b) Using bounded variable technique, solve the following LPP: (5)

$$\max Z = 6x + 2y + 3z$$

subject to $-2x + 4y + 2z \leq 9$, $-x - 2y + 3z \leq 10$

$$0 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 2$$

2. Find the optimal solution of following bounded variable transportation problem to minimize the transportation cost: (10)

| | | | | availability |
|-------------|----|----|----|--------------|
| | 3 | 7 | 6 | 15 |
| Cost | 2 | 4 | 3 | 22 |
| | 4 | 3 | 8 | 23 |
| Requirement | 20 | 15 | 25 | |

such that $x_{ij} \geq 0$ and the upper bounds are given in the following matrix:

$$[u_{ij}] = \begin{bmatrix} 12 & 15 & 20 \\ 13 & 22 & 10 \\ 10 & 11 & 12 \end{bmatrix}$$

3. Use wolfe's method to solve the quadratic programming: (10)

$$\min Z = x^2 - xy + 2y^2 - x - y$$

subject to $2x + y \leq 1$ $x, y \geq 0$.

4. (a) Check whether following problem is a convex programming problem and verify the result graphically: (5)

$$\max y$$

subject to $x^2 + y^2 \leq 1$, $x^2 \geq y$.

- (b) Let for each $i = 1, 2, \dots, m$, g_i be a convex function. Then prove that the set S is a convex set: (5)

$$S = \{x \in R^n : g_i(x) \leq 0, (i = 1, 2, \dots, m)\}$$

5. Using Lagrange multiplier method, find the extreme values of $f(x, y, z) = xy + z^2$ on the on the circle in which the plane $y - x = 0$ intersects the sphere $x^2 + y^2 + z^2 = 4$. (10)
6. (a) Let $f(x)$ be differentiable at $x^* \in S$ and $\nabla f(x^*) \neq 0$. Then $\hat{d} = -\nabla f(x^*)$ is a usable feasible direction at x^* provided \hat{d} is a feasible direction. (5)

(b) Consider the NLP: $\max Z = \ln(1+x) + y$ subject

$$2x + y \leq 3, \quad x, y \geq 0$$

Write the KKT conditions. Given that the optimal solution of the above NLP lies on the line $y = 3$, use KKT conditions to find its optimal solution. (5)

7. Using conjugate gradient method find the solution of the system: (10)

$$x + y = 8, \quad 2x + y = 10$$

You may take the starting point as $x^{(0)} = (0, 0)^T$.

8. (a) Solve the following linear fractional programming problem by the simplex algorithm (6)

$$\max Z = \frac{2x + y}{x - 2y}$$

subject to

$$x + y \leq 6, \quad 2x + y \geq 4, \quad x, y \geq 0$$

- (b) Write the Wolfe dual of the following NLP: (4)

$$\max Z = 2x + 4y$$

subject to $x^2 + y^2 \leq 5, \quad x - y \leq 2$.

X - X - X