1st + 2nd Sem.

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**Exam.Code:0905**
**Sub. Code: 6641**

**1108**
**B.E. (Bio-Technology) First Semester**
**MATHS-101: Calculus**
**(Common to all streams)**

**Time allowed: 3 Hours** **Max. Marks: 50**

***NOTE:*** *Attempt five questions in all, including Question No. I which is compulsory and selecting two questions from each Part.*

*x-x-x*

I. Attempt the following:-

a) Define sequence and limit of a sequence.. Give an example of a sequence which is bounded but not convergent.

b) Define convergence of an alternating series. Also define absolute convergence and conditional convergence with suitable examples.

c) If $x = r\cos\theta$, $y = e\sin\theta$, find $\frac{\partial x}{\partial r}$ *and* $\frac{\partial r}{\partial x}$.

d) Explain the difference between scalar point function and vector point function. Also give one example of each.

e) How does divergence theorem genalizes Green's theorem? Also give their physical interpretation. (5x2)

## UNIT – I

II. a) Examine the convergence or divergence of the sequences (i) $a_n = n - \sqrt{n^2 - n}$ (ii) $a_n = l_{nn} - l_{n\,(n+1)}$

b) Determine how many terms should be used to estimate the sum of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{(n+3\sqrt{n})^3}$ with an error less than 0.0001.

c) Check the convergence or divergence of the following series:

i) $\sum_{n=1}^{\infty}\frac{(-1)^n(2n)!}{2^n n!n}$

ii) $\sum_{n=1}^{\infty}\frac{1.3.5.....(2n-1)}{[2.4.6......(2n)](3^n+1)}$

iii) $\sum_{n=1}^{\infty}\frac{n!\ln n}{n(n+2)!}$ (2,4,4)

III. a) Find the volume of the solid generated by revolving the region enclosed by (1,0), (2,1) and (1,1) about the y-axis.

b) Let $w = f(u) + g(v)$, where $u = x + iy$ and $v = x - iy$, and $i = \sqrt{-1}$. Show that w satisfies the Laplace equation $w_{xx} + w_{yy} = 0$, if all the necessary functions are differentiable. (5,5)

IV. a) The temperature at a point (x, y) on a metal plate is $T_{(x,y)}=4x^2 - 4xy + y^2$. An out on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the out.

P.T.O.

b) Evaluate $\iiint |xyz|\, dxdydz$ over the solid ellipsoid : $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\le 1$. (5,5)

## UNIT – II

V. a) Find the areas of the region common to interiors of cardioids $r = 1 + \cos\theta$ and $r = 1 = \cos\theta$

b) Use Green's theorem to find counter-clockwise circulation and outward flux for the field $\vec{F} = (y^2 - x^2)\hat{i} + (x^2 + y^2)\hat{j}$ over the curve C the triangle bounded by $y = 0$, $x = 3$ and $y = x$. (5,5)

VI. a) Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plan $z = 4$.

b) Find unit tangent vector (T), unit normal vector (N), unit bionormal vector (B), curvature (K) and torsion ($\tau$) for the space curve $\vec{r}(t) = e^t \cos t\,\hat{i} + e^t \sin t\,\hat{j} + 2\hat{k}$. (5,5)

VII a) Verify Stoke's theorem for the vector find $\vec{F} = (3x - y)\hat{i} - 2yz^2\hat{j} - 2y^2z\,\hat{k}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$, $z > 0$.

b) Prove that the curvature of a smooth curve $\vec{r}(t) = f(t)\hat{i} + g(t)\hat{j}$ defined by twice differentiable functions $x = f(t)$ and $y = g(t)$ is given by the formula

$$K = \frac{|xy - yx|}{(x^2 + y^2)^{\frac{3}{2}}}$$

Apply this formula to find the curvature of $\vec{r}(t) = [\tan^{-1}(\cosh t)\, i + l_n(\sinh t)\, j]$. (5,5)

x-x-x