Exam. Code: 0918
Sub. Code: 6793

1058

# B. Engg. (Computer Science and Engineering)
## $6^{th}$ Semester
## CS-602: Linear Algebra and Probability Theory

Time allowed: 3 Hours Max. Marks: 50

*NOTE:* *Attempt five questions in all, including Q. No. I which is compulsory and selecting atleast two questions from each Unit. Use of statistical table and non-programmable calculator is allowed.*

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I. (a) Let $S = \{(-1,0,1), (2,1,4)\}$. Find the values of k for which the vector $(3k+2,3,10)$ belongs to a linear if an of S.

(b) Let V be a 3-dimensional vector space over the field F of 3 elements. Then, what is the number of distinct 1-dimensional sub-spaces of V?

(c) Give an example of an infinite dimensional vector space.

(d) Show that if A is idempotent matrix, then all the eigenvalues of A are equal to 0 or I.

(e) Find the eigenvalues and eigen vectors of the matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

(f) State central limit theorem.

(g) If a random variable X assume, the value 0 and 1 with $P = (X = 0) = 3P(X = 1)$ then find the variance of X?

(h) Compute the correlation coefficient between random variables X and X if $V(X) = V(Y) = \frac{1}{4}$ and $V(X - Y) = \frac{1}{3}$.

(i) If $\lambda$ is a Poisson variable such that $P(\lambda = 2) = 9P(\lambda = 4) + 90P(\lambda = 6)$, then find the mean of $\lambda$.

(j) What is normal distribution? Draw a rough sketch of its probability density function and describe its two important properties. (10×1)

## UNIT-I

II. (a) Find the values of 'k' such that the system of equations: $x + ky + 3z = 0, 4x + 3y + kz = 0, 2x + y + 2z = 0$ has non-trivial solution.

(b) Let $\mu$ be the vector space of all 3×3 real matrices and let $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

Then, prove that $W = \{x \in M : trace(A\lambda) = 0\}$ is a subspace of $\mu$. (5+5)

III. (a) Let W be the subspace of $R^4$ spanned by the vectors $\vec{V_1} = (1,-2,5,-3)$, $\vec{V_2} = (2,3,1,-4)$, $\vec{V_3} = (3,8,-3,-5)$:

(i) Find the basis and dimension of W.

(ii) Expand the basis of W to a basis of $R^4$.

**P.T.O**

(b) Let $B_1 = \{(1,2),(2,-1)\}$ and $B_2 = \{(1,0),(0,1)\}$ be ordered basis of $R^2$. If $T : R^2 \to R^2$ is a linear transformation such that $[T]_{B_1,B_2}$, the matrix of T with respect to bases $B_1$ and $B_2$ is $\begin{bmatrix} 4 & 3 \\ 2 & -4 \end{bmatrix}$. Then, find the value of T(5+5). (5+5)

IV. (a) Check whether the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$ in diagonalizable or not? If yes, then diagonalize it.

(b) State rank-nullity theorem. Find the basis and dimension of null space and image space of the transformation $T : R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2, x_1 - x_2, 0)$. Also, verify rank-nullity theorem. (5+5)

## UNIT-II

V. (a) A die is tossed twice. Getting a number greater than 4 is considered as a success. Find the mean and variance of the probability distribution of number of successes.

(b) Using Poisson distribution, find the probability that at most 5 defective bolts will be found in a box of 200 bolts if it is known that 2% of such bolts are expected to be defective (Take $e^{-4} = 0.0183$). (5+5)

VI. (a) State Chebyshev inequality. What is the approximate value of $P(1\lambda - \mu |\leq 2\sigma)$ when we use Chebyshev inequality. Here, x denotes the continuous random variable having the following probability density function $f(x) = \begin{cases} 630 \quad x^4(1-x)^4, & if\, 0 < x < 1 \\ 0 & otherwise \end{cases}$

(b) A restaurant serves two special dishes A and B to its customer consisting of 60% men and 40% women. 80% of men order dish A and the rest B. 7% of women order B and the rest A. In what ratio of A to B should the restaurant prepare the two dishes? (5+5)

VII. (a) Explain the relationship between binomial and normal distribution.

(b) In a normal distribution, 31% of items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution. Also, find out what % of the items differ from the mean by a number not more than 5? (5+5)

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