

Sub. Code: 6842

1058

B.E. (Electronics and Communication Engineering)
Fourth Semester
MATHS-405: Linear Algebra and Complex Analysis

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Section.

x-x-x

1. (A). Let $V = \mathbb{R}^3$ and (i) $W = \{(a, b, c); a, b \in \mathbb{R}\}$ then Show that W is a subspace of V.
(ii) $W = \{(a, b, c); a+b+c=0\}$ then Show that W is a subspace of V.
(iii) $W = \{(a, b, c); a \geq 0\}$ then Show that W is not a subspace of V. (1+1)
- (B). Determine whether or not the following vectors in \mathbb{R}^3 are linearly dependent:
 $u = (1, -2, 1), v = (2, 1, -1), w = (7, -4, 1)$ (1)
- (C). Show that similar matrices have the same eigen values. (2)
- (D) State Residue's Theorem. (1)
- (E). Define conformal and Isogonal mappings with examples. (1+1)
- (F). Show that the function $\frac{z^2 - \bar{z}^2}{zz}$ does not have a limit as z tends to 0. (2)

SECTION A

2. Let V be a finite dimensional vector space and $F: V \rightarrow U$ be a linear transformation. Then prove that
$$\dim V = \text{rank}(F) + \text{Nullity}(f)$$
 (10)
3. Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be the linear mapping defined by $T(x, y, z) = (x+2y-z, y+z, x+y-2z)$
Find a basis and the dimension of the (i) Image of U of T (ii) Kernel W of T. (5+5)
4. State and prove Cayley – Hamilton's Theorem. (10)

SECTION B

5. Find the bilinear transformation which maps the points $z_1 = 1, z_2 = i, z_3 = -1$ onto the points $w_1 = i, w_2 = 0, w_3 = -i$. For this transformation, find the image of (i) $|z| \leq 1$ (ii) Concentric circle $|z| > r$ ($r > 1$). (6+2+2)
6. (i) State necessary and sufficient conditions for a function $f(z)$ of a complex variable to be analytic at a point. (2)
(ii) Evaluate $\oint_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C is the circle: $|z-i|=2$ (8)
7. Expand the function $f(z) = \frac{1}{z(z^2-3z+2)}$ in Laurent series for the regions
(a) $0 < |z| < 1$; (b) $1 < |z| < 2$; (c) $|z| > 2$ (4+4+2)

x-x-x