

1048
B.E., Second Semester
AS-201: Engineering Mathematics – II
(May – 2014 Common)

Time allowed: 3 Hours

Max. Marks: 50

NOTE: Attempt five questions in all, including Question No. 1 which is compulsory and selecting two questions from each Part. Use of programmable calculator is allowed.

x-x-x

PART A

1. Solve the following differential equations: (3+3+4)

(a) $x dy = (y + x^2 + 9y^2)dx$

(b) $(1 + x^2)dy + 2xy dx = \cot(x) dx$

(c) $(D^6 - 2D^3 + 1)y = e^{2x}$ where $D \equiv \frac{d}{dx}$

2. (a) Find the general solution of the differential equation: (5)

$$(D^2 + 1)y = x^2 e^{2x} + \cos(2x)$$

- (b) Solve the differential equation using Laplace transforms technique: (5)

$$(D^3 - 3D^2 + 3D - 1)y = t^2 e^t, y(0) = 1, y'(0) = 0, y''(0) = -2$$

3. (a) State and prove convolution theorem for Laplace transform. (5)

- (b) Using the method of variation of parameters solve the following differential equation: (5)

$$(D^2 + 4D + 4)y = \frac{2e^{-2x}}{x^2}$$

4. (a) Evaluate the integral $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t} dt$. (3)

- (b) Find the Laplace transform of $f(t) = e^{-4t} \int_0^t \frac{\sin(3u)}{u} du$. (4)

- (c) Find the inverse Laplace transform of $\ln \frac{s+1}{s-1}$. (3)

PART B

5. (a) Find the general solution of the partial differential equation: (7)

$$(x - y)y^2 p + (y - x)x^2 q = (x^2 + y^2)z$$

- (b) Formulate the partial differential equation by eliminating the arbitrary function: (3)

$$f(x^2 + y^2 + z^2, z^2 - 2xy) = 0$$

6. (a) Find the fourier series of the **periodic function** $f(x)$ with period $p = 2\pi$ defined below: (5)

$$f(x) = \begin{cases} \pi e^{-x} & \text{if } -\pi < x < 0 \\ \pi e^x & \text{if } 0 < x < \pi \end{cases}$$

- (b) Prove that

$$\int_0^{\infty} \frac{\sin(\pi w) \sin(xw)}{1-w^2} dw = \begin{cases} \pi \sin(x)/2 & \text{if } 0 \leq x \leq \pi \\ 0 & \text{if } x > \pi \end{cases} \quad (5)$$

7. (a) Find the fourier sine transform of $f(x) = e^{-\pi x}$. (6)

- (b) Find the trigonometric polynomial approximation of degree $N = 2$ for the function $f(x) = x^2$ ($-\pi < x < \pi$) for which the total square error with respect to this function is minimum on the interval $[-\pi, \pi]$ (4)

8. Find the solution of the one dimensional wave equation for an elastic string of length L which is fixed at the end points with initial deflection and initial velocity as $f(x)$ and $g(x)$ respectively. (10)

x-x-x