Exam. Code: 0939
Sub. Code: 7045

1078
B. Engg. (Mechanical Engg.)
3rd Semester
AS-301: MATHS-3

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt five questions in all, including Q. No. 1 which is compulsory and selecting atleast two questions from each Unit.*

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I. (a) Define bounded sequence and monotonic sequences with suitable examples. Examine the same for the sequences: (i) $a_n = \left\{\frac{1}{2^n}\right\}$ (ii) $a_n = \{(-1)^n\}$

(b) State Taylor's series for the approximation of a function. Explain how the error is estimated using it.

(c) Define eigenvalue problem of the matrices. Write down any two applications of this problem. Obtain the eigenvalues and eigenvectors for the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(d) Examine whether or not w=cos z is bounded or not? Justify your answer.

(e) Define conformal and isogonal mapping with suitable examples. Also define critical points of a mapping. (5×2)

UNIT-I

II. (a) Examine which of the following sequences $\{a_n\}$ converge, and which diverge? Find the limit of each convergent sequence:

(i) $a_n = n - \sqrt{n^2 - n}$ (ii) $a_n = \frac{1}{\sqrt{n^2 - 1} - \sqrt{n^2 + n}}$

(iii) $a_n = \left(\frac{n}{n+1}\right)^n$ (iv) $a_n = \left(\frac{3n+1}{3n-1}\right)^n$

(b) State and prove Cauchy's integral test. Hence, examine the convergence of the p-series: $\sum_{n=1}^{\infty} \frac{1}{n^p}$ (5+5)

III. (a) For the following power series, find radius and interval of convergence. For what values of x does the series converge (i) absolutely, (ii) conditionally?

(i) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n!}$ (ii) $\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2}$

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(b) Describe the principle involved in the Gauss elimination method. Solve the system: $x+10y-z=3$; $2x+3y+20z=7$ *and* $10x-y+2z=4$ using it with partial pivoting. (5+5)

IV. (a) Examine whether the matrix $A=\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$ in diagonalizable? If so, obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix.

(b) Verify Cayley-Hamilton theorem for the matrix $A=\begin{bmatrix}-1 & 2 & 0\\ -1 & 1 & 2\\ 1 & 2 & 1\end{bmatrix}$. Also find A^{-1}. (5+5)

UNIT-II

V. (a) Examine the confinity of the function $f(z)=\begin{cases}\dfrac{\text{Im}(z)}{|z|} & , \quad z\neq 0\\ 0 & , \quad z\neq 0\end{cases}$ about the point z=0.

(b) Show that the function $f(z)=\bar{z}$ is continuous at the point z=0 but not differentiable at z=0.

(c) Prove that the function w=cos z is analytic in the finite z-plane. (3+4+3)

VI. (a) Prove that the function $v(x,y)=e^{x}\sin y$ is harmonic. Find its conjugate harmonic function u(x,y) and the corresponding function f(z).

(b) Find all the Taylor's and Laurent series expansion of $f(z)=\dfrac{1}{z^2-1}$ about $z_0=1$. (5+5)

VII. (a) Evaluate the integral: $I=\int_0^{2\pi}\dfrac{d\theta}{2+\sin\theta}$

(b) Find the bilinear transformation which maps 1,i,-1 to 2, i,-2 respectively. Find the fixed and critical points of the mapping. (5+5)

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