Exam. Code: 0933
Sub. Code: 6974

1078

B. Engg. (Electrical & Electronics Engg.)

3^{rd} Semester

MATHS-301: Linear Algebra and Complex Analysis

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt five questions in all, including Q. No. 1 which is compulsory and selecting atleast two questions from each Unit. Use of simple calculator is allowed.*

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I. (a) Define linear span. Determine the span of each of the following rectors in R^3:

(i) $S_1 = \{(1,0,0)\}$ (ii) $S_2 = \{(1,0,0),(0,1,0)\}$

(b) Show that intersection of two subspaces of a vector space in a subspace.

(c) Define singular and non-singular linear maps. Determine whether or not the linear map given by $T: R^2 \to R^2$ defined by $T(x, y) = (x - y, x - 2y)$ is singular or non-singular. If non-singular, then find T^{-1}.

(d) Define harmonic functions with suitable example. Write down their applications.

(e) Define conformal and isogonal mapping with suitable examples. (5×2)

UNIT-I

II. (a) Solve the system of equations:

$x + 2y - 2z = 1,$

$2x - 3y + z = 0$

$5x + y - 5z = 1$

$3x + 14y - 12z = 5$ by Gauss elimination method.

(b) Define basis of a vector space. Prove that the subset s of R^3 is a spanning set of R^3, but not a basis of R^3: $S = \{(2,2,3),(-1-2,1),(0,1,0)\}$ (5+5)

III. (a) If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a linear polynomial in A. Also, verify Cayley-Hamilton theorem and compute A^{-1}.

(b) Examine whether the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ in diagonalizable or not? If diagonalizable, then diagonalize it. (5+5)

P.T.O.

IV. (a) Let $T : R^3 \to R^3$ be the linear transformation defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis and the dimensions of (i) the image of T, (ii) the null space of T.

(b) Consider the linear transformation T on R^2 defined by $T(x, y) = (5x - y, 2x + y)$ and the following bases of R^2:

$E = \{\bar{e}_1, \bar{e}_2\} = \{(1,0), (0,1)\}$

$S = \{\bar{u}_1, \bar{u}_2\} = \{(1,4), (2,7)\}$

(i) Find the change of basis matrix P from E to S and the change of basis matrix Q from S back to E.

(ii) Find the matrix A in at representing T in the basis E.

(iii) Find the matrix B that represents T in the basis S. (5+5)

UNIT-II

V. (a) Define analytic function with a suitable example. If $u(x, y) = e^{-x}(x \sin y - y \cos y)$ then form analytic function $w = u(x, y) + i(\cos y)$,

(b) (i) Find all values of z such that $\sinh z = e \dfrac{\pi i}{3}$

(ii) Examine whether the function $f(z) = \begin{cases} \dfrac{\mathrm{Re}(z^2)}{|z|^2} & , \ z \neq 0 \\ 0 & , \ z = 0 \end{cases}$

is continuous at z=0 or not? (5+5)

VI. (a) Expand $f(z) = \dfrac{z}{(z-1)(2-z)}$ in Laurent series valid for:

(i) $|z - 1| > 1$ (ii) $0 < |z - 2| < 1$

(b) Define three types of isolatetd singularities with an example for each. (5+5)

VII. (a) Evaluate $\int_0^{2\pi} \dfrac{\cos 2\theta}{5 + 4\cos\theta} d\theta$ using residue theorem of integration.

(b) Discuss the mapping $w = z + \dfrac{1}{z}$. (5+5)

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