Exam. Code: 0907
Sub. Code: 6699

1078

B. Engg. (Bio-Technology)-3rd Semester

AS-301: Engg. Mathematics-III

(Common)

Time allowed: 3 Hours Max. Marks: 50

NOTE: *Attempt five questions in all, including Q. No. 1 which is compulsory and selecting atleast two questions from each Unit.*

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I. (a) Define limit of a sequence. Prove that limits of sequences are unique.

(b) Prove that if $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n \to \infty} a_n = 0$ but the converse is not true.

(c) Describe the principle involved in the Gauss-Jordan method for finding the inverse of a square matrix A.

(d) Find all values of z which satisfy $e^z = 1 + i$.

(e) Classify the singular point z=0 of the function: $f(z) = \dfrac{e^z}{z - \sin z}$ (5×2)

UNIT-I

II. (a) Find all the values of z such that $\sin z = 2$.

(b) Show that the function $f(z) = |z|^2$ in differentiable only at z=0 and no where else.

(c) Prove that $w = \sin z$ is analytic in the finite z-plane. (4+3+3)

III. (a) If f(z) is analytic in a domain D and $|f(z)|$ in a non-zero constant in D, then prove that f(z) is constant in D.

(b) Find all possible Taylor's and Laurent series expansion for the function: $f(z) = \dfrac{1}{z(z-1)}$ about z_0=0. (5+5)

IV. (a) State residue theorem. Evaluate the integral: $I = \oint_C \dfrac{dz}{(z-1)(z-2)^2}$, *where* $C: |z-2| = \dfrac{1}{2}$

(b) Evaluate the integral: $I = \int_0^{2\pi} \dfrac{d\theta}{a + \cos\theta}, a > 1$ (5+5)

P.T.O.

UNIT-II

V. (a) Examine which of the following sequences converge, and which diverge? Find the limit of each convergent sequence:

(i) $a_n = \dfrac{\sin n}{n}$ (ii) $a_n = \dfrac{(\ln n)^{100}}{n}$

(iii) $a_n = \tan^{-1} n$ (iv) $a_n = \dfrac{(\ln n)}{n - \frac{1}{n}}$

(b) Examine the convergence or divergence of the following series:

(i) $\sum_{n=0}^{\infty} \dfrac{\cos n\pi}{5^n}$, (ii) $\sum_{n=0}^{\infty} \dfrac{2}{1+e^n}$

(iii) $\sum_{n=0}^{\infty} \dfrac{3}{n+\sqrt{n}}$ (iv) $\sum_{n=0}^{\infty} \dfrac{(-100)^n}{n!}$ (5+5)

VI. (a) Find the radius and interval of convergence for the series: $\sum_{n=1}^{\infty} \dfrac{(4x-5)^{2n+1}}{n^{\frac{3}{2}}}$.

For what values of x does the series converge:

(i) absolutely, (ii) conditionally

(b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$, using the Gauss-Jordan method with partial pivoting. (5+5)

VII. (a) Show that the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is diagonalizable. Find P such that $P^{-1}AP$ in diagonal matrix.

(b) Verify Caylley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ and hence find A^{-1}. (5+5)

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