Exam. Code: 0905
Sub. Code: 6641

1078

B. Engg. (Bio Technology)-1st Semester

MATHS-101: Calculus

***(Common to all streams)***

Time allowed: 3 Hours

Max. Marks: 50

***NOTE:*** *Attempt five questions in all, including Q. No. 1 which is compulsory and selecting atleast two questions from each Unit.*

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I. (a) Define absolute and conditional convergence of a series with suitable examples.

(b) Transform the equation: $\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into polar coordinates.

(c) Explain the physical significance of curl, gradient and divergence with suitable examples.

(d) State Green's and Stoke's theorem.

(e) Calculate the angle between the normals to the surface $xy=z^2$ at the points (4,1,2) and (3,3,-3). (5×2)

**UNIT-I**

II. (a) Which of the following sequences $\{a_n\}$ converge, and which diverge? Find the limit of each convergent sequence:

(i) $a_n = n - \sqrt{n^2 - n}$ (ii) $a_n = \frac{1}{\sqrt{n^2-1} - \sqrt{n^2+n}}$

(iii) $a_n = \frac{(\ln n)^6}{\sqrt{n}}$ (iv) $a_n = \left(\frac{x^n}{2n+1}\right)^{\frac{1}{n}}, x > 0$

(b) Show that the p-series: $\sum_{n=1}^{\infty} \frac{1}{{}_n p} = \frac{1}{{}_1 p} + \frac{1}{{}_2 p} + \frac{1}{{}_3 p} + \ldots\ldots + \frac{1}{{}_n p} + \ldots.$ (p a real constant) converges if $p > 1$ and diverges if $p \le 1$. (5+5)

III. (a) Discuss the convergence or divergence of the following series:

(i) $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$ (ii) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$ (iii) $\sum_{n=1}^{\infty} \frac{4^n n! n!}{(2n)!}$

(b) Find the area of the surface generated by revolving the curve $y = x^2, 0 \le x \le \frac{1}{2}$ about the x-axis. (5+5)

**P.T.O.**

IV. (a) If $x+y=2e^{\theta}\cos\varphi$ *and* $x-y=(2e^{\theta}\sin\varphi)i$ show that $\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^2 u}{\partial\varphi^2}=4xy\frac{\partial^2 u}{\partial x\partial y}$.

(b) Find the volume of the greatest rectangular parallels piped that can be inscribed in the ellipsoid: $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ (5+5)

**UNIT-II**

V. (a) Evaluate $\iint r^3 dr d\theta$ over the area bounded between in circles $r=2\cos\theta$ *and* $r=4\cos\theta$

(b) Find the directional derivative of $\varphi=x^2yz+4xz^2$ at a point (1, -2, 1) in the direction of the vector $2\hat{i}-\hat{j}-2\hat{k}$. (5+5)

VI. (a) Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing to polar co-ordinate. Hence, show that $\int_0^\infty e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$

(b) Show that for the curve: $\vec{r}(t)=a(3t-t^3)\hat{i}+3at^2\hat{j}+a(3t+t^2)\hat{k}$, the curvature equals torsion. (5+5)

VII. (a) Evaluate $\int_C (xy+z^2)$ when C is the arc of the helix $x=\cos t,\quad y=\sin t,\quad z=t$ which joints the points $(1,0,0)$ *and* $(-1,0,\pi)$.

(b) Verify Green's theorem for $\int_C (3x-8y^2)dx+(4y-6xy)dy$ where C is the boundary of the region bounded by x=0, y=0 and x+y=1. (5+5)

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